EFFECTS OF WINDS AND OF BAROMETRIC PRESSURES ON THE GREAT LAKES

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EFFECTS OF WINDS AND OF BAROMETRIC PRESSURES ON THE GREAT LAKES.

GENERAL INTRODUCTION.

The investigation of wind effects and barometric effects* on the Great Lakes, which is the subject of this publication, is a part of a much larger investigation, covering a much broader field, which has been in progress since the summer of 1911 except as interrupted by the Great War. The ultimate object of the larger investigation is to obtain a much better formulation than the engineering profession now has of the laws governing the amount of stream-flow.

Whenever it is proposed to use the water which is being delivered by a river, for the development of power, for irrigation, or for the supply of a city or town, one is confronted with the desirability of predicting the amount of the future flow of that river and its variations. It is necessary to know, as a basis for the best design of the proposed works, not only the mean flow of the stream, but usually also its maximum flow, its minimum flow, and in general the characteristic features of its variation in flow. The more accurately these things can be predicted the better the design of the works may be made with respect to economy and safety. The decision, in the first place, as to whether the proposed works are worth while frequently turns upon the predicted future flow of the stream. So, too, when it is proposed to construct great engineering works for the prevention of damage from floods, such as those now under construction in the Miami Conservancy District, for the protection of Dayton, Ohio, and other nearby towns, one needs at the outset accurate predictions of the future flow in the streams concerned, especially of the amount of the flood flows and their duration. Any substantial improvement in the possibilities of accurate prediction of the future flow of streams would be of great value to the engineers and to humanity.

It appeared to be probable that progress toward this ultimate object, the securing of a better formulation of the laws governing the amount of stream-flow, is conditioned upon first securing a better knowledge than is now available of the laws of evaporation from large water surfaces, such as the surfaces of lakes and rivers, and from land surfaces.

To secure a better understanding of the laws of evaporation, it seemed to be of first importance to supplement the numerous and intensive studies

* Throughout this paper the effects of barometric pressures are termed barometric effects.

of evaporation, which have been made on a small scale by using evaporation pans with an area of a few square feet or a few square inches, by an investigation made on the full scale of nature and under natural conditions. For this purpose it was proposed to consider each of the Great Lakes in turn as an evaporation pan and to evaluate from day to day (1) the change of content, (2) the income, and (3) the outgo, including evaporation. There is hope that these evaluations can be made with sufficient accuracy to segregate that part of the outgo which is evaporation, and to determine the laws which control the rate of evaporation. If that hope is realized, the laws so determined may be applied with a confidence based upon the fact that the experimental evidence is on the full scale of nature and under natural conditions.

The change of content of any one of the Great Lakes from day to day is measured by the change of elevation of the mean lake surface from day to day. The area of the lake surface remains substantially constant. For Lake Erie, for example, the area never varies by as much as 0.01 part from 9,968 square miles. If the elevation of the mean surface of the lake is observed to have increased by 0.01 foot, the content of the lake is known to have increased by the net addition of a sheet of water 0.01 foot thick having an area of 9,968 square miles. Such a sheet contains about 2,800,000,000 cubic feet and is equivalent to the outflow through the Niagara River under typical conditions for about 3.7 hours—at 210,000 cubic feet per second.

The elevation of the surface of each of the Great Lakes has been determined at a few selected points by recording gages operated continuously day and night for many years by the United States Lake Survey. The most important gages are at Tibbetts Point (New York), Buffalo (New York), Cleveland (Ohio), Harbor Beach (Michigan), Mackinaw City (Michigan), Milwaukee (Wisconsin), and Marquette (Michigan). As the investigation progressed, it gradually became more clearly evident that the largest and most serious errors encountered were those which arise from the fact that the surface of any one of the Great Lakes at any given instant is not level except by accident. The surface has a slope at every point due to the influence of winds and barometric pressures. Hence, when the elevation of the surface of Lake Erie, for example, is observed at Buffalo on each day, it is necessary, if one is to obtain daily values of the elevation of the mean surface of Lake Erie, to apply a correction each day to correct for the disturbance of the elevation on that day at Buffalo produced by the winds and barometric pressures.

The recording gages are so designed and operated that the rapid fluctuations of elevation of the water surface which one sees as wind waves are practically eliminated from the record. The elevation recorded by the gage at any instant is with a high degree of accuracy, usually within 0.1 or even within 0.01 foot, equal to the mean elevation for the preceding 10 minutes, even though the wind waves may be causing the instantaneous elevation of the water at the gage to vary through a range of several feet several times in

each minute. The influence of the slopes and differences of elevation visible to the eye as waves are eliminated automatically, and with an exceedingly high degree of accuracy, by the gages. Such slopes are local in character, they extend continuously without change of sign for a few feet only, and at any one spot on the lake surface they persist of one sign for a few seconds only. The mean elevation of the water surface as recorded by a gage is not appreciably affected by these rapid fluctuations of slope and elevation.

Omit from consideration the rapid and large fluctuations of slope and elevation of short periods referred to in the preceding paragraph. Just as the effects of these visible wind waves are automatically eliminated, by the gages, from the record written by them, so imagine the surface of the lake as a whole to be freed from the visible wind waves, and imagine each part of the surface to be at its mean elevation—a mean covering say 10 to 15 minutes. This smoothed-out lake surface on any one of the Great Lakes will not at any time be level except by accident. It will always be disturbed by wind and by barometric pressures. But the slope in that surface will always and everywhere be too small to be visible to the unaided eye. Anywhere on the Great Lakes where the depth of water is as much as 10 feet and under the most extreme conditions of wind and barometric pressure such slopes are probably not greater than 0.011 foot vertical to one mile horizontal. The slopes are usually less than 0.002 foot vertical to one mile horizontal wherever the depth of water is 20 feet or more; they are, however, as a rule continuous with one sign over the whole surface of the lake at any instant, though at times there is one reversal of sign in passing from one end (or side) of a lake to the other. That is, in general, if the slope produced by the wind and barometric pressures combined is downward from Buffalo to the westward at a given instant, it is downward to the westward continuously all the way to the west end of the lake. Occasionally, and for short periods only, the slope of the character referred to may be downward to the westward in a part of Lake Erie and upward to the westward in the remaining part, or vice versa. The slopes of the character referred to in this paragraph persist with one sign for many hours, as a rule, and sometimes for several days, until the wind changes or the barometric conditions change.

It is these long-continuous, invisible slopes of the water surface, produced by winds and differences of barometric pressures, that cause the recorded mean elevation, for any hour or any day, at a gage, to be different from the actual mean elevation of the whole surface of the lake for that hour or that day. The difference between the mean elevation of the water surface at a gage for any hour or any day and the mean elevation of the whole lake surface during that same time is the combined wind effect and barometric effect. This publication deals with the investigation of these effects of winds and of barometric pressures. It shows how the investigation has been made and how corrections may be applied to eliminate the major part of the effects of winds and of barometric pressures, and so to secure much more accurate values than are otherwise obtainable for the mean elevation of

the whole surface of each lake. The manner in which the conclusions reached in this investigation may be applied to other lakes or bodies of water and may help in solving various problems is briefly indicated.

DATA USED AND ACKNOWLEDGMENTS.

No new observations were made in this investigation. The necessary observations, in great abundance, and of adequate accuracy and reliability, had already been made. The principal data used include hourly and daily observed elevations of the surface of Lake Erie and Lake Michigan–Huron at five gage stations, observed hourly wind directions and velocities at five points near these two lakes, and the observed barometric pressures twice per day at six points.

The hourly elevations of the water surface which were used on Lake Erie covered 48 selected days at Buffalo and 52 selected days at Cleveland. On Lake Michigan–Huron the hourly elevations of the water surface which were used covered 34 days at Milwaukee, 42 days at Harbor Beach, and 42 days at Mackinaw.

The daily mean elevations of the water surface which were used on Lake Erie covered the months August–October 1909 and June–October 1910—eight months in all. Each daily mean was the mean of 24 hourly values for that day obtained from the records of the automatic gages which had operated at Buffalo and Cleveland. The two sets of daily means were entirely separate and independent for the two stations Buffalo and Cleveland.

On Lake Michigan–Huron the daily mean elevations of the water surface which were used were separate and independent series at each of the three stations, Milwaukee, Mackinaw, and Harbor Beach, each covering the months June–September 1910 and June–September 1911—eight months in all. Each daily mean was the mean of 24 hourly values for that day obtained from an automatic gage.

The observed hourly winds which were used had been obtained from recording anemometers and wind vanes as operated at the U. S. Weather Bureau stations at Buffalo, Cleveland, Milwaukee, Port Huron (Michigan) and Sault Ste. Marie (Michigan). For each hour the total travel of wind in that hour was observed and the prevailing direction to the nearest of the principal points, N, NE, E, SE, etc., at intervals of 45°. The hourly winds used at the stations named cover all of the months named in connection with the daily mean elevations of water surface and also all of the separate days for which hourly water elevations were used.

The observed barometric pressures at six stations near the two lakes were read from the isobars (lines of equal barometric pressure) as shown on the daily forecast maps as prepared at the U. S. Weather Bureau at Chicago for the use of the forecaster at that station. Two such maps are made for each day, one showing the facts at 8 a.m., 75th meridian time, and the other facts at 8 p.m., 75th meridian time. The barometric pressures used in this investigation were for the two times named on each of the dates covered by the

daily mean elevations of the water surface and by the hourly elevations of the water surface which have been mentioned.

The above statement shows what data were fully used in the portion of the investigation covered specifically by this publication. Many more data of these kinds have been collected and used to furnish general checks on the conclusions reached. For example, many more daily mean elevations of water surface at various stations were collected and examined. So, too, the barometric pressures have been secured and examined at eleven stations in the vicinity of the Great Lakes for the two times named above on each day in 28 months during the years 1909 to 1913, inclusive.

Grateful acknowledgment is hereby extended to the two organizations which have furnished the data used in this investigation and to several persons in those organizations who have been especially helpful in supplying information and valuable suggestions.

The United States Weather Bureau has furnished all meteorological data promptly or has given the opportunity to get it from the original records. The Chief of the Weather Bureau, C. F. Marvin, has taken a continuous interest in the investigation, has supplied many comments and suggestions, and has given freely the benefit of his good judgment. Prof. Henry J. Cox, meteorologist, in charge of the Weather Bureau station at Chicago, has freely supplied working facilities during many days at various times for those who have been working on this investigation. He and his assistants have facilitated the work in many ways, but especially by granting free access to the forecast maps and other records at the Chicago office.

The United States Lake Survey (Survey of the Northern and Northwestern Lakes) has furnished freely a large amount of information in regard to hourly and mean daily elevations of the water surface at various gage stations on the Great Lakes, of which that used in the particular investigation treated in this publication is but a part.

Acknowledgment is hereby made to the various officers of the Corps of Engineers of the United States Army who have been in charge of the Lake Survey at the various times when data have been requested and furnished. Especial acknowledgment is also extended to Mr. F. G. Ray and to Mr. Thomas Russell, of the Lake Survey, for information given and courtesies extended at various times in connection with this investigation. Mr. Walter J. Graves, formerly of the Lake Survey, also furnished especially valuable information and suggestions early in the investigation.

Mr. J. A. Folse worked on this investigation as a computer for 4,000 hours in all, intermittently, with many interruptions in 1913–20, and continuously from June 1920 to July 1921, inclusive. In 1920 and 1921 all phases of the investigation were discussed fully with him, and he has incidentally given many valuable suggestions. The extensive use of plotted data, studied in the graphic form, to supplement the analyses made by the least-square method of computation, has been due largely to his insistent suggestions. Such use has proved to be very effective in leading to a true understanding of certain phases of the investigation.

METHODS OF THIS INVESTIGATION.

In general, both in this investigation of wind effects and barometric effects and in the larger investigation of evaporation from the Great Lakes, of which it is a part, the procedure has been as indicated in the following numbered paragraphs:

(1) The immediate problem to be attacked was studied in the light of all available information. A theory was developed as to the relations of the various quantities involved. The theory was expressed in the form of a general equation. The equations were then set up for a least-square solution to test that theory. Each equation expressed an observed quantity, a change in elevation of the water surface, in terms of other known or observed quantities, in conformity with the theory which was on trial. The solution was then made by the least-square method of computation. The principal outcome was (*a*) a set of computed values of the unknowns, assumed to be constants, which were supposed to express the relations between the observed quantities, and (*b*) a set of residuals which are the discrepancies between the tentative theory and the observed facts.

(2) The outcome of the least-square solution was then studied in the light of all available internal and external evidence. As the theory expressed in the observation equations of such a solution approaches more closely to perfection and to completeness, the computed probable errors are smaller, the residuals as a rule are smaller, and the distribution of the residuals as to sign and magnitude follow the laws of accidental errors more closely. These tests furnished the main portion of the internal evidence. The external evidence was derived mainly (*a*) from comparisons of the outcome of the solution with that from other solutions already made, (*b*) from a study of apparently abnormal residuals, and (*c*) from general checks on the reliability of the various items of outcome of the solution which were derived from any available information which was independent of the least-square solutions.

(3) In some cases, especially in the later portions of the investigation, the observed fluctuations in elevation of the water surface were plotted in graphs, and the fluctuations as computed from the constants derived from the least-square solutions were plotted, superposed, on the same scale. The graphs were then studied to secure further checks, contradictions, or suggestions.

(4) In the light of all evidence a new set of observation equations was then set up and the whole process repeated of making a least-square solution and then studying the outcome from it. In general, each new set of observation equations involved one or more of the following: (*a*) a change in the tentative theory, expressed as a change in the form of the observation equations; (*b*) a change in the data used, brought about by rejecting certain observation equations or by combining certain others (two or more in a group) to form one; (*c*) the addition of a considerable amount of data, so as to increase the number of observation equations to at least double what it had been, or (*d*)

the new observation equations were based upon an entirely independent group of data, elevations of water surface, from a different gage, which might even be on a different lake.

Though the general procedure was that indicated above, the arrangements were such, especially when several computers were working at the same time, that two or more least-square computations were in progress simultaneously, relating to different gages or to different phases of the investigation. In general, the investigation has been so managed that each general conclusion adopted depends upon two or more least-square solutions and the corresponding studies based upon independent sets of data, usually from different gages and if feasible from different lakes.

The least-square method of solution was adopted as the principal method of attack on this problem because it was apparent that several different factors or influences were operating simultaneously to cause fluctuations of the elevation of the water surface at a gage, no one of which could be safely neglected while attempting to evaluate others. In this investigation, dealing with the facts on the full scale of nature and under uncontrolled natural conditions, it is not feasible to use the familiar laboratory strategy of eliminating from the phenomena by control the influence of all factors except one, determining the influence of that one, and then doing likewise for each of the other factors in turn. Hence, it was believed that the best method of attack in this case is the least-square method of computation. That method is especially adapted to taking into account simultaneously several controlling factors and to determining simultaneously their separate influences. The outcome has fully justified this method.

In the broader investigation of evaporation, including the investigation of wind effects and barometric effects which is here reported, 74 complete least-square solutions and the corresponding studies have been made. Typical solutions each contained from 100 to 600 observation equations, each having for its absolute term the observed change in elevation of the water surface at a gage during a day or an hour, and each containing from 2 to 8 unknown constants to be determined. In an extreme solution each observation equation contained 40 unknowns, and there were 619 observation equations in the set.

Of the 74 least-square solutions referred to in the preceding paragraph, 9 were directly utilized in determining the wind effects and barometric effects. All of the remainder contributed more or less indirectly by gradually leading the investigator toward a true understanding of these effects and therefore toward a true expression of the theory.

The statement just made, indicating the large number of least-square solutions made and their complexity, shows this to have been an extensive investigation. More than 22,500 man-hours of time have been spent on the routine part of the computations and studies connected with the broad investigation of evaporation, including the investigation of wind and barometric effects. From 1 to 10 persons have been engaged in this routine work

at any one time at various intervals since it was commenced in the summer of 1911. In all, 31 persons have taken part. I have spent more than 2,000 hours on the investigation, none in routine work.

OUTCOME OF THE INVESTIGATION.

The outcome of the investigation may be very briefly characterized as follows:

(1) Reasonably accurate numerical expressions have been obtained for the effects of barometric pressures on the elevation of the water surface at the five stations, Buffalo, Cleveland, Milwaukee, Mackinaw, and Harbor Beach, on Lake Erie and on Lake Michigan–Huron. With these expressions, one may, from the distribution of barometric pressures ordinarily shown on the forecast maps of the Weather Bureau, compute the disturbances in elevation of the water surface thereby produced at the stations named.

(2) The general method has been developed by which such a numerical expression for the barometric effect at any station on any body of water may be derived from observations of the water elevation at that station and the forecast maps for the same period.

(3) A general expression, including the necessary numerical constant, has been obtained for the effect of winds, of any given velocity and direction, in producing a disturbance of elevation of the water surface at any given station, on any body of water, anywhere in the world. The data required in regard to the station and the body of water are such as are ordinarily shown on good charts, namely, the depths of the water at all points, the location of the shore line, and the location of the station.

(4) Four of the prevailing seiches, or free oscillations under the influence of inertia, on Lake Erie and Lake Michigan–Huron have been isolated. Their periods and probable methods of oscillation have been shown. The relation between these seiches and the uncertainties in daily mean elevations of the water surface at gage stations has been discerned. The appreciation of this relation aids decidedly in obtaining accurate determinations of the daily mean elevation of the mean surface of each lake.

(5) The accuracy with which the elevation of the mean surface of any one of the Great Lakes may be determined for any given day has been decidedly increased. On Lake Erie the elevation of the mean surface of the lake may now be determined as accurately from one day of observation at Buffalo as it was formerly possible to fix it from 16 days of observation at that station. Similarly, the elevation of the mean surface of Lake Michigan–Huron may now be determined as accurately from one day of observation at Mackinaw as it was formerly possible to determine it from 6 days of observation at that station. When one determines the fluctuation of elevation of the mean surface of a lake he thereby determines the fluctuation in the total water-content of the lake.

(6) The relations of the new knowledge indicated in (1) to (5) to four outstanding problems have become evident. The four problems are:

(*a*) The problem of regulating the elevations of the water surface of each of the Great Lakes—and the rates of flow through the connecting streams, so as to secure the greatest aggregate benefits to navigation, power, development, and sanitation.

(*b*) The problem of determining the laws of evaporation from large free-water surfaces such as the surface of the Great Lakes.

(*c*) The problem of correcting the observed elevations of the water-surface at a tide-gage in such a manner as to remove the disturbances due to winds and fluctuating barometric pressures and thereby to secure a more accurate determination of mean sea-level than could otherwise be obtained from said observations.

(*d*) The problem of determining the direction and rate of the tilting, which is believed to be in progress, of the land underlying and immediately surrounding the Great Lakes.

ORDER OF PRESENTATION.

The order of presentation in this publication is briefly indicated in the following paragraphs:

(1) The manner in which the least-square solutions which serve to determine barometric effects were set up is first given, including the theoretical basis of the observation equations. The principal facts in regard to the five final barometric solutions are given, including the values computed from them. The manner of using these values to compute hourly and daily barometric effects is set forth.

(2) Similarly, the manner of setting up the least-square solutions which serve to determine the wind effects is given, together with the principal facts in regard to the four final wind solutions and the values computed from them. The formula, constants, and method for computing hourly wind effects are set forth. The method used in computing daily wind effects is shown.

(3) The accuracy of the computed barometric effects is discussed, using both the internal evidence of the computations and external evidence.

(4) Similarly, the accuracy of the computed wind effects is discussed.

(5) The evidence as to the over-all accuracy attained in the attempt to secure elevations of the mean surface of a whole lake by applying corrections for wind effects and barometric effects at the gage stations is discussed.

(6) The evidence obtained in regard to seiches, free oscillations under the influence of inertia, in Lake Erie and Lake Michigan–Huron, is set forth and discussed.

(7) Certain generalizations are made as to wind effects, barometric effects, and seiches. These generalizations are intended to help one, during a first reconnaissance of a problem connected with any lake (or other large body of water), to form a first approximate estimate as to the probable magnitude and character of the wind effects, barometric effects, and seiches on that lake and their probable bearing on the problem to be attacked.

(8) The relation of the research to four important outstanding problems in science and engineering is briefly indicated.

The table of contents at the beginning of this publication is intended to give a good general view of the order of presentation in more detail than the preceding statement; it will assist especially the reader who is referring back to the publication to look up a particular topic.

THEORETICAL BASIS FOR BAROMETRIC-OBSERVATION EQUATIONS.

What is the shape of the surface of a lake when its water is in equilibrium under the influence of gravity and barometric pressures? The wind is ignored in this question. The answer is evidently the desired fundamental theoretical basis for a study of barometric effects—disturbances of elevation of the water surface at a gage station produced by barometric pressures.

If there is equilibrium at every part of the lake, under the conditions stated, with no wind blowing, according to the fundamental principle of hydrostatics the pressure at every point in the lake at a given elevation must be the same as at every other point at that elevation. Also that pressure must be at every point, x,

$$p=(H_s-H_x)\delta_w+M\delta_m \tag{1}$$

in which

H_s is the elevation of that part of the surface of the water which is directly above the point.

H_x is the elevation of the point.

δ_w is the density of the water.

M is the length of the mercury column which measures the barometric pressure upon the surface of the water above the point.

δ_m is the density of mercury.

H_s-H_x is the distance from the surface of the water down to point X.

$(H_s-H_x)\delta_w$ is the pressure at the point X due to the weight of the water above it.

$M\delta_m$ is the pressure at the point X due to barometric pressure at the surface above, which pressure is necessarily transmitted to the point X under the conditions stated.

Note that M is the ordinary expression, incorrectly used, for the barometric pressure, namely, the length of mercury column which will balance the barometric pressure, say 30 inches, under certain conditions.

Consider the relations between the quantities H_s and M for two points, 1 and 2, at the same elevation H_x in the water of the lake, under the conditions of equilibrium which have been stated. Let the elevation of the water surface be called H_1 at point 1 and H_2 at point 2. Let the pressures at the two points be called p_1 and p_2, respectively. Let the barometric pressures as ordinarily expressed, in terms of M, at the water surface above the two points be called M_1 and M_2, respectively.

For the condition of equilibrium $p_1 = p_2$ and therefore from equation (1) it is clear that

$$(H_1 - H_x)\delta_w + M_1\delta_m = (H_2 - H_x)\delta_w + M_2\delta_m \tag{2}$$

From equation (2) it follows, by cancellation and rearrangement of terms, that

$$H_1 - H_2 = -(M_1 - M_2)\frac{\delta_m}{\delta_w} \tag{3}$$

Equation (3) expresses the fact that under conditions of equilibrium the contour lines (lines of equal elevation) on the surface of the water must coincide in shape with the isobars (lines of equal barometric pressure at the surface of the water), that increasing elevations from contour to contour must correspond to decreasing barometric pressures from isobar to isobar, and that a unit interval between contours must correspond to an interval of $\frac{\delta_m}{\delta_w}$ between isobars.

The elevation of the water surface on the Great Lakes is ordinarily expressed in feet above mean sea-level. $H_1 - H_2$ is therefore most conveniently expressed in feet. The barometric pressures are usually expressed by the U. S. Weather Bureau in inches of mercury at 0°C., and were so recorded on the forecast maps used in this investigation. The density of mercury at 0°C. = 13.6. The water concerned in equation (3) is the surface water of the lakes, of which the temperature will seldom be outside the limits 32°F. = 0°C. and 80°F. = 27°C. Therefore the density of this water will seldom be outside the extreme limits 1.000, at 39°F. = 4°C. and 0.997, at 80°F. = 27°C. With sufficient accuracy for the present purpose the density of the water may be assumed to be constant at 1.00, and $\frac{\delta_m}{\delta_w}$ may therefore be assumed to be constant at 13.6.

For convenience, recognizing the units ordinarily used for elevations and for barometric pressures, equation (3) may now be rewritten thus

$$H_1 - H_2 = -(M_1 - M_2)(13.6)(\tfrac{1}{12}) = -(M_1 - M_2)(1.13) \tag{4}$$

Note that the division by 12 in the second member of the equation is to take into account the fact that the M's are expressed in inches whereas the H's are expressed in feet.

The upper part of plate 1 shows the isobars over Lake Erie and vicinity as obtained from the forecast map of the U. S. Weather Bureau as used at the Chicago office of the Bureau for 8 p.m. on August 5, 1910. Note that the interval between isobars is 0.01 inch, that the isobars are nearly straight across Lake Erie, and that the barometric gradient is downward to the northeastward over Lake Erie.

The middle part of plate 1 shows the contours of the surface of the water on Lake Erie at 8 p.m. on August 5, 1910, in accordance with equation (4). These contours would have existed if no wind had been blowing at that time and if the water of Lake Erie had been in equilibrium at that time under the influence of gravity and barometric pressure. Note that an arbitrary zero

has been adopted for these contours coinciding with the location of the isobar marked 29.90. Note, also, that the interval between contours is 0.0113 foot, which corresponds, in accordance with equation (4), to the interval of 0.01 inch between the isobars shown on the upper part of the plate. Note, that the surface gradient of the water of Lake Erie is shown as upward to the northeastward in accordance with the barometric gradient shown on the upper part of the plate as downward to the northeastward.

Plate 1 is a concrete illustration of the relation between isobars and surface contours at any water surface which must exist if the water is in equilibrium under the influence of gravity and barometric pressure.

BAROMETRIC EFFECTS IN TERMS OF BAROMETRIC PRESSURES.

In setting up observation equations to express the relation between observed fluctuations in water elevation and barometric pressures as shown on the forecast maps it would be desirable to utilize accurately the relations shown in equation (4) and illustrated in plate 1, if limits of time, expense, and accuracy did not prevent it from being feasible. But a reconnaissance of the problem (including an attempt, on a small scale, to utilize the exact relations) indicated that in order to keep within limits imposed on this investigation it was necessary to introduce an assumption, called assumption No. 1.

Assumption No. 1.

It is assumed that, with sufficient accuracy for the purposes of this investigation, the isobars at the instant represented by any forecast map (8 a.m. or 8 p.m., 75th meridian time) are straight and uniformly spaced within the limits of the lake under consideration.

Assumption No. 1 is nearly true for the actual case illustrated by plate 1. In general, for each of the Great Lakes assumption No. 1 is nearly true. The size of any one barometric low-pressure area or of any high-pressure area is usually many times as great as that of any one of the lakes. Within the area occupied by the lake the isobars curve but little and there is but moderate departure from uniform spacing. The most serious departures from assumption No. 1 in these two respects ordinarily occur when the center of a well-developed low-pressure area is over a lake. One may verify these statements by studying the forecast maps.

Assumption No. 1 was introduced to save time and expense. The ultimate effect of its introduction in reducing the accuracy of the final computed barometric effects is believed to be moderate only.

Assumption No. 1 combined with the relations between isobars and contours on the water surface which have been commented upon and which are fixed by equation (4) makes the contours on the water surface straight and uniformly spaced. In other words, under assumption No. 1, whenever the water is in equilibrium under the influence of gravity and barometric pressures its surface is plane.

In the case illustrated in plate 1, for the actual isobars there shown one

must now, under assumption No. 1, substitute other isobars coinciding as nearly with the actual isobars as is consistent with the condition that they must everywhere over Lake Erie be straight and uniformly spaced. The corresponding contours on the lake surface, for the conditions of equilibrium, will be straight and uniformly spaced as indicated in the lower part of plate 1.

The total amount of water in Lake Erie is not affected by the barometric pressure, or its distribution. The effect of the greater barometric pressure on the southwestern part of the lake as compared with that on the northeastern part of the lake is to subtract water from the southwestern part and add it to the northeastern part. Some line on the surface of the water, such as that marked as the nodal line on the lower part of Plate 1, is not changed in elevation. What is the location of that line? The direction of the nodal line is evidently parallel to the contours. It remains to fix one point on the nodal line.

Consider an elementary portion of the lake surface as shown on the lower sketch on plate 1 of which the two dimensions are δL at right angles to the contours and δW parallel to the contours, of which the area is $\delta L \delta W = \delta A$ and of which the distance from the nodal line is L. Let the slope of the water surface be called S.

The volume of water which had been added at this area by the barometric influence is

$$\text{Depth of added water (area)} = SL\delta L\delta W = SL\delta A$$

SL is the depth of water added at the particular area.

The total amount of water added to the northeastward of the nodal line is the integral over that portion of lake of these elementary volumes, namely:

$$\int SL\delta L\delta W = \int SL\delta A = S\int L\delta A \qquad (5)$$

S may be placed outside the integral sign, as it is the slope which has been assumed to be constant.

Consider the portion of the lake which lies to the southwestward of the nodal line, from every portion of which water is subtracted by the barometric influence. Use the same notation as before, but let L be counted as negative when measured to the southwestward from the nodal line. Then for this portion of the lake the total amount of water subtracted is the same integral as before, namely,

$$S\int L\delta A \qquad (6)$$

in which, however, all values of L are negative and the integral is negative.

As the total amount of water in the lake has not been changed, the sum of integrals (5) and (6) must be zero—that is, the amount of water added on one side of the nodal line must equal that subtracted from the other side. In other words the integral $S\int L\delta A$ over the whole of the lake surface must be zero, the distance L being reckoned from the nodal line as indicated. Hence, S being a constant, $\int L\delta A$ over the whole area of the lake must be zero.

The well-known condition which locates the so-called center of gravity of an area is that the integral over the whole area $\int L\delta A$ is zero with L reckoned from any line through the center of gravity. Hence, it is clear that the nodal line under assumption No. 1 of isobars which are straight and uniformly spaced is a line parallel to the isobars passing through the center of gravity of the area of the lake surface.

The position of the center of gravity of the area of Lake Erie is indicated in the lower part of plate 1 by the circle labeled "C. G. of lake area." It is 634,000 feet west and 276,000 feet south of the Buffalo gage station. Its latitude is 42° 07′ and its longitude is 81° 13′.

For all conditions of equilibrium under the influence of gravity and barometric pressures, under the restrictions of assumption No. 1, the elevation of the lake surface at the center of gravity remains unchanged.

Let H_c be this fixed elevation of the water surface at center of gravity—that is, fixed and unchangeable in so far as changes of barometric pressure limited by assumption No. 1 are concerned. Let M_c be the barometric pressure on the surface of the water at the center of gravity. Then equation (4) may be rewritten thus:

$$E_1 = H_1 - H_c = -(M_1 - M_c)(1.13) \tag{7}$$

in which E_1 is the barometric effect, under the specific conditions, on the elevation of the water surface at the point 1.

For convenience in computation it is now proposed to express $H_1 - H_c$ in terms of slopes of the water surface measured along parallels and meridians and to express $M_1 - M_c$ similarly in terms of barometric gradients measured along parallels and meridians.

The barometric gradient between two points is the difference in barometric pressures at the two points divided by the distance between the points.

Let the barometric gradient along a parallel be called the "*W–E* gradient," and let it be called positive when the barometric pressure increases to the westward. Similarly, let the barometric gradient along a meridian be called the "*N–S* gradient," and let it be called positive when the barometric pressure increases to the northward.

Let the co-ordinates of point 1 measured from the center of gravity of the lake along parallels and meridians be L_w along a parallel and L_n along a meridian as indicated on the lower part of plate 1. Let L_w be considered positive to the eastward and L_n positive to the southward.

Then, keeping in mind that under assumption No. 1 the isobars are straight and equally spaced,

$$M_1 - M_c = -(W\text{–}E \text{ gradient})(\mathrm{L}_w) - (\text{N–S gradient})(L_n) \tag{8}$$

Similarly,

$$H_1 - H_c = -(\text{W–E slope})(L_w) - (\text{N–S slope})(L_n) \tag{9}$$

in which the slope of the water surface along a parallel is called the "W–E slope," positive when it is upward to the westward, and the slope of the

water surface along a meridian is called the "N–S slope," positive when it is upward to the northward. Equation (9) is true, because the water surface under assumption No. 1 is an inclined plane.

By substitution in equation (7) of the values of $M_1 - M_c$ and of $H_1 - H_c$ from equations (8) and (9) there is obtained

$$E_1 = -(\text{W–E slope})(L_w) - (\text{N–S slope})(L_n)$$
$$= +(\text{W–E gradient})(L_w)(1.13) + (\text{N–S gradient})(L_n)(1.13) \qquad (10)$$

Or, dropping out the middle part of (10), which is no longer necessary after the conceptions are grasped

$$E_1 = +(\text{W–E gradient})(L_w)(1.13) + (\text{N–S gradient})(L_n)(1.13) \qquad (11)$$

EXPRESSION OF BAROMETRIC GRADIENTS.

From a study of the forecast maps and of the general conditions of the problem, it was decided that the best feasible way to secure satisfactory values for the W–E gradient and the N–S gradient for each of the Great Lakes at the many times for which they were needed was as follows:

(1) Eleven points were selected for which readings were to be taken from the forecast maps to cover all of the Great Lakes region. The six of these points used in connection with Lake Erie and Lake Michigan–Huron, known as points 3, 4, 5, 6, 7, and 8, are shown on the sketch on the lower half of plate 2. Their exact locations are shown in the adjoining table:

Point.	Lat.	Long.
3	47½°	85°
4	45	87½
5	45	80
6	42½	85
7	40	80
8	42½	77½

(2) The values of the barometric pressure were read directly from the forecast maps for 8 a.m. and 8 p.m. (75th meridian time) of each day and tabulated in convenient form.

(3) Let "(6–8)" stand for the barometric pressure at point 6 minus the barometric pressure at point 8 for a given time. Let "(distance 6 to 8)" stand for the distance from point 6 to point 8. Note, that points 6 and 8 are on the same parallel, that they are both in the latitude of Lake Erie, point 6 to the westward and point 8 to the eastward. The (W–E gradient) for Lake Erie was then taken as

$$\frac{(6\text{–}8)}{(\text{distance 6 to 8})} \qquad (12)$$

(4) Similarly, the (N–S gradient) for Lake Erie was taken as

$$\frac{(5\text{–}7)}{(\text{distance 5 to 7})} \qquad (13)$$

(5) For Lake Michigan–Huron, the (W–E gradient) was taken as

$$\frac{(4\text{–}5)}{(\text{distance 4 to 5})} \qquad (14)$$

and the (N–S gradient) was taken as

$$\frac{(3\text{–}6)}{(\text{distance 3 to 6})} \tag{15}$$

The procedure outlined above involves assumption No. 2, which is stated in the following paragraph:

Assumption No. 2.

It is assumed that the barometric gradients at any time on any lake along parallels and meridians are the same as the barometric gradients derived, as indicated above, from readings taken from the forecast maps at the selected points 3, 4, 5, 6, 7, and 8, which lie on meridians and parallels through the lakes.

It is believed that this assumption is only a fairly good approximation, that the barometric gradients over the shorter distances limited by the lake surface vary through a larger range than the gradients over the longer distances between the reading points used on the forecast maps, and that the variations of barometric gradients over the shorter and longer distances may not be in step—that is, one may in general be ahead of the other. It is believed, however, that the errors in assumption No. 2 are largely canceled out in so far as the final values of the computed barometric effects are concerned. This cancellation is believed to be affected in part by the device of introducing the proportionality factors P_w and P_n into the derivation of the observation equations as indicated later, and in part by the process of deducing, from the results of the least-square computations, the lag of barometric effects behind the changes in barometric gradients.

For Lake Erie the expression for E_1, the barometric effect at any point 1 on that lake, as shown in equation (11), may now be rewritten as follows by means of expressions (12) and (13):

$$E_1 = +(6\text{–}8)\left(\frac{L_w}{\text{distance 6 to 8}}\right)(1.13) + (5\text{–}7)\left(\frac{L_n}{\text{distance 5 to 7}}\right)(1.13)$$
$$= +(6\text{–}8)R_w + (5\text{–}7)R_n \tag{16}$$

in which

$$R_w = \frac{1.13\ L_w}{\text{distance 6 to 8}} \quad \text{and} \quad R_n = \frac{1.13\ L_n}{\text{distance 5 to 7}} \tag{17}$$

R_w and R_n are constants for any given point on Lake Erie. They differ for different points but do not change with the lapse of time.

PROPORTIONALITY FACTORS FOR BAROMETRIC EFFECTS.

Equation (16) expressed the barometric effect on the elevation of the water surface at any given point 1 on Lake Erie provided the water always remained in equilibrium under the influence of gravity and barometric pressure. The general conception thus far set forth, and including assumption No. 1, is that the barometric pressures over Lake Erie are continually

changing but always in such manner that the isobars over the lake are straight and uniformly spaced, and that the fluctuations in the elevations of different parts of the water surface take place at once in such a manner that the water is always in equilibrium.

It is obvious that the effects of friction and of inertia will tend to modify the response of the water to changing barometric pressures.

Friction will tend in general to reduce the range of fluctuation of water surface and to produce a lag of the response behind the barometric changes which produce it.

Inertia will tend to produce an initial lag in the response of the water surface to any change in the barometric pressure. But when the water has once started from one part of the lake toward another, as the water surface is approaching a new condition of equilibrium after some relatively sudden change of barometric gradients, inertia will tend to carry the water past the position of equilibrium and thereby to make the fluctuations of elevation of the water surface greater than those corresponding to continuous equilibrium.

If friction is relatively large, so that all motions of the water produced by inertia, all free oscillations, are quickly damped out, the fluctuations in the elevation of the water surface at any point would tend to be considerably less in range than those which would be computed from equation (16). On the other hand, if friction is relatively ineffective in damping out free oscillations of the water of the lake under the influence of inertia, and if the natural periods of oscillation of the lake happen to bear certain relations to the periods of change in the barometric gradients, the actual fluctuations in the elevation of the water surface at a point might largely exceed those computed from equation (16).

Hence, aside from providing later for an assumed lag to be determined by the observations themselves, through the least-square solution, it is also advisable to introduce into equation (16) proportionality factors P_w and P_n, to be determined from the observations.

Hence, equation (16) is now rewritten thus for Lake Erie:

$$E_1 = +(6\text{–}8)R_wP_w + (5\text{–}7)R_nP_n = +(6\text{–}8)C_w + (5\text{–}7)C_n \qquad (18)$$

in which P_w and P_n are proportionality factors not necessarily assumed to be equal, and

$$C_w = R_wP_w \text{ and } C_n = R_nP_n \qquad (19)$$

It is desirable to note that the proportionality factors P_w and P_n, to be derived from the observations, tend to take into account several effects: (1) certain effects arising from friction and free oscillations just referred to; (2) errors of certain kinds in assumption No. 2, to which attention has already been called (on page 16). There may also be some tendency for the wave produced by barometric influences to be modified by the configuration of the shores and bottom as it progresses in such wise that the wave may be accentuated or modified and given a larger, or possibly smaller, range at the gage station than it otherwise would have. This will also be taken into

account (in part at least) by the proportionality factors P_w and P_n derived separately from the observations at each gage station. The accentuation or modification here referred to is that peculiar to the particular locality at and near the gage station, and which is likely to exist in addition to the general accentuation or modification of the wave considered as one unit for a whole lake.

FORM OF OBSERVATION EQUATIONS FOR BAROMETRIC EFFECTS.

It is desirable to express the relation between the mean barometric effects at any point on Lake Erie on two successive days, on the one hand, and the barometric difference (6–8) and (5–7), see pages 15–16, on the other hand. The development of the corresponding expression for Lake Michigan–Huron will be given later.

It is proposed to write one observation equation for each day. The day to which the equation is credited will be called the current day and the next earlier day will be called the preceding day. Each equation is to express the change in the barometric effect from the preceding to the current day.

Let it be assumed that between 8 a.m. and 8 p.m. of the preceding day (6–8) increased by an amount $-b_{w_1}=$ (6–8) at 8 p.m. minus (6–8) at 8 a.m., at a uniform rate, and that no other changes in barometric gradients occurred in the two days.

From equation (18), on the assumptions stated and assuming that there is no lag, the increase in the elevation of the water surface at the gage station will be at a uniform rate from 8 a.m. to 8 p.m. and the total rise will be $b_{w_1}C_w$. The variation in elevation of water surface at the gage will be expressed by the line marked "B1 No Lag" on plate 3. Counting from the dotted zero line indicated on the drawing, the elevation of the water will be zero during the 8 hours from midnight to 8 a.m. on the preceding day, and will vary from zero to $b_{w_1}C_w$ during the 12 hours from 8 a.m. to 8 p.m. with a mean elevation of 0.5 $b_{w_1}C_w$ during that 12 hours. The elevation of the water surface will remain $b_{w_1}C_w$ during the last 4 hours of the preceding day and throughout all of the 24 hours of the current day.

With reference to the dotted line the mean elevation of the water surface on the preceding day will therefore be

$$\frac{(0.5\,b_{w_1}C_w)(12)+(b_{w_1}C_w)4}{24}=\frac{10}{24}b_{w_1}C_w$$

and on the current day will be $b_{w_1}C_w$.

Hence, the increase in mean elevation for the current day over the mean elevation for the preceding day will be

$$b_{w_1}C_w-\frac{10}{24}b_{w_1}C_w=\frac{14}{24}b_{w_1}C_w=b_{w_1}B_{w_1} \tag{20}$$

in which

$$B_{w_1}=\frac{14}{24}C_w \tag{21}$$

Similarly, the line marked "B_2 No Lag" in plate 3 represents the variation in the elevation of the water surface which will occur if (6–8) increases by an amount $-b_{w_2}$ at a uniform rate from 8 p.m. of the preceding day to 8 a.m. of the current day. By the same process of reasoning which was used above in connection with b_{w_1}, it may be shown that the increase in mean elevation for the current day over the mean elevation for the preceding day will be

$$\frac{21.3}{24} b_{w_2} C_w - \frac{.7}{24} b_{w_2} C_w = \frac{20.7}{24} b_{w_2} C_w = b_{w_2} B_{w_2} \tag{22}$$

in which

$$B_{w_2} = \frac{20.7}{24} C \tag{23}$$

So, too, the line marked "B_3 No Lag" on plate 3 represents the variation in elevation if (6–8) increases by an amount $-b_{w_3}$ at a uniform rate from 8 a.m. of the current day to 8 p.m. of the current day. In this case the increase in the mean elevation for the current day over the mean elevation for the preceding day will be

$$\frac{10}{24} b_{w_3} C_w = b_{w_3} B_{w_3} \tag{24}$$

in which

$$B_{w_3} = \frac{10}{24} C_w \tag{25}$$

The line marked "B_0 No Lag" on plate 3 represents the variation in elevation if (6–8) increases an amount $-b_{w_0}$ at a uniform rate from 8 p.m. of the day before the preceding day to 8 a.m. of the preceding day. In this case the increase in the mean elevation for the current day over the mean elevation for the preceding day will be

$$\frac{2.7}{24} b_{w_0} C_w = b_{w_0} B_{w_0} \tag{26}$$

in which

$$B_{w_0} = \frac{2.7}{24} C_w \tag{27}$$

Note that B_{w_0}, B_{w_1}, B_{w_2}, and B_{w_3} (see equations 27, 21, 23, and 25) are all constants to be derived from the least-square computation. Each one involves the proportionality factor P_w and other values which are different at different stations but do not vary with the lapse of time. Consult equations (19) and (17).

Normally, (6–8) is found to be different at each of the successive epochs 8 a.m. and 8 p.m. at which the barometric pressures are determined from the forecast maps. Hence, on the assumptions which have been stated, the total decrease, due to this cause, in the mean elevation of the current day over the mean elevation for the preceding day will be

$$b_{w_0} B_{w_0} + b_{w_1} B_{w_1} + b_{w_2} B_{w_2} + b_{w_3} B_{w_3} \tag{28}$$

Let the corresponding notation with reference to barometric gradients along the meridian be used. That is, let b_{n0}, b_{n1}, b_{n2}, and b_{n3} be the amounts by which (5–7) decreases in each of the several 12-hour periods which were specified in connection with b_{w0}, b_{w1}, b_{w2}, and b_{w3}, respectively; and let B_{n0}, B_{n1}, B_{n2}, and B_{n3} have meanings corresponding to those already specified for B_{w0}, B_{w1}, B_{w2}, and B_{w3}, respectively. By the same reasoning which was used in connection with barometric gradients along parallels, it may be shown that the decrease, due to change in barometric gradients along meridians, in the mean elevation of the current day over the mean elevation of the preceding day will be

$$b_{n0}B_{n0}+b_{n1}B_{n1}+b_{n2}B_{n2}+b_{n3}B_{n3} \tag{29}$$

On the assumptions thus far stated, the form of each observation equation, one for each day, is as follows:

$$b_{w0}B_{w0}+b_{w1}B_{w1}+b_{w2}B_{w2}+b_{w3}B_{w3}+b_{n0}B_{n0}+b_{n1}B_{n1}+b_{n2}B_{n2}+b_{n3}B_{n3}+I=V \tag{30}$$

In equation (30), I is the observed rise in the elevation of the water surface at the gage station—that is, the mean observed elevation for the current day minus the mean observed elevation for the preceding day. The second member of the equation is a residual, v, which is the discrepancy between theory and observation for the particular day. The least-square solution serves to determine the most probable values for the unknowns B_{w0}, B_{w1} . . . B_{n2}, B_{n3}. These values are the ones which will make the sum of the squares of the system of residuals, v, a minimum.

Note that B_{n0}, B_{n1} . . . B_{n2}, B_{n3} have values in terms of C_w and C_n which are tabulated in the third line of table No. 1 (page 21) as derived from equations (27), (21), (23), and (25). When these values of B_{w0}, B_{w1} . . . B_{n2}, B_{n3} have been determined by the least-square solution, it will then be possible to compute C_w and C_n.

LAG IN BAROMETRIC EFFECTS.

Thus far, in fixing the form of the observation equations (30), it has been assumed that there is no lag in the response of the water to barometric changes.

Let it be assumed that the response occurs with a lag which is to be determined from the observations. The observation equations will remain as before, as shown in (30), but certain modifications, which are about to be indicated, will be necessary in interpreting the equations and in interpreting the derived values of B_{w0}, B_{w1}, . . . B_{n2}, B_{n3}.

Let it be assumed for a moment that the response of the water surface to barometric changes lags 4 hours behind such changes. Then the responses of the water surface are properly represented on the same basis as before by the lines marked "B_1, 4^{h}Lag," "B_2, 4^{h}Lag," "B_3, 4^{h}Lag," and "B_0, 4^{h}Lag" on plate 3.

Consider in detail the first of these cases. It is assumed in this case that between 8 a.m. and 8 p.m. of the preceding day (6–8) increased by an amount

$-b_{w_1}=(6-8)$ at 8 p.m. minus (6–8) at 8 a.m., at a uniform rate, that no other changes in barometric gradients occurred in the two days, and that the lag of the change in the water surface behind the barometric change is 4 hours. The increase in the elevation of the water surface at the gage station will be at a uniform rate from noon to midnight. The total rise will be $b_{w_1}C_w$. The rise is properly shown in the line on plate 3 marked "B_1, 4^hLag." Counting from the dotted zero line indicated on the drawing, the elevation of the water will be zero during the 12 hours from midnight to noon of the preceding day and will vary from zero to $b_{w_1}C_w$ during the 12 hours from noon to midnight on the preceding day, with a mean elevation of 0.5 $b_{w_1}C_w$ during that 12 hours. The elevation of the water surface will remain $b_{w_1}C_w$ throughout all of the 24 hours of the current day. With reference to the dotted line, the mean elevation of the water surface on the preceding day will therefore be

$$\frac{(0.5\, b_{w_1}C_w)12}{24}=\frac{6}{24}b_{w_1}C_w$$

and on the current day will be $b_{w_1}C_w$. Hence, the increase in mean elevation of the current day over the preceding day will be

$$b_{w_1}C_w-\frac{6}{24}b_{w_1}C_w=\frac{18}{24}b_{w_1}C_w=b_{w_1}B_{w_1} \tag{31}$$

in which

$$B_{w_1}=\frac{18}{24}C_w \tag{32}$$

TABLE No. 1.

	B_{w_0} or B_{n_0}	B_{w_1} or B_{n_1}	B_{w_2} or B_{n_2}	B_{w_3} or B_{n_3}
Anticipation, or negative lag, 2 hours	1.5	12.0	21.0	12.0
Anticipation, or negative lag, 1 hour	2.0	13.0	20.9	11.0
No lag	2.7	14.0	20.7	10.0
Lag, 1 hour	3.4	15.0	20.2	9.0
Lag, 2 hours	4.2	16.0	19.6	8.0
Lag, 3 hours	5.1	17.0	18.9	7.0
Lag, 4 hours	6.0	18.0	18.0	6.0
Lag, 5 hours	7.0	18.9	17.0	5.1
Lag, 6 hours	8.0	19.6	16.0	4.2
Lag, 7 hours	9.0	20.2	15.0	3.4
Lag, 8 hours	10.0	20.6	14.0	2.7
Lag, 9 hours	11.0	20.9	13.0	2.1
Lag, 10 hours	12.0	21.0	12.0	1.5
Lag, 11 hours	13.0	21.0	11.0	1.0
Lag, 12 hours	14.0	20.7	10.0	.7
Lag, 13 hours	15.0	20.3	9.0	.4
Lag, 14 hours	16.0	19.7	8.0	.2
Lag, 15 hours	17.0	19.0	7.0	.0
Lag, 16 hours	18.0	18.0	6.0	.0
Lag, 17 hours	18.9	17.0	5.1	——

Similarly, by following through the reasoning corresponding to lines "B_2, 4^hLag," "B_3, 4^hLag," and "B_0, 4^hLag" on plate 3 it may be shown that

$$B_{w_2}=\frac{18}{24}C_w \tag{33}$$

$$B_{w_3}=\frac{6}{24}C_w \tag{34}$$

$$B_{w_0}=\frac{6}{24}C_w \tag{35}$$

when the lag is 4 hours.

The various values of B_{w_1}, B_{w_2}, . . . corresponding to a lag of 4 hours are shown in the seventh line of table No. 1.

The tabular values are B_{w_0} or B_{n_0}, etc., in terms of C_w or C_n expressed in units of $\frac{1}{24}$ C_w or $\frac{1}{24}$ C_n.

The various lines in table No. 1, for various assumed values of the lag, have each been computed by the method illustrated by the examples which have been given.

Table No. 1 makes it clear that the values of B_{w_0}, B_{w_1}, . . . B_{n_2}, B_{n_3}, depend upon the actual lags and upon the actual values of C_w and C_n. If observation equations of the form shown in (30) are used and the values of B_{w_0}, B_{w_1}, . . . B_{n_2}, B_{n_3} are derived from the least-square solution, the lag may be determined from these values by the use of table No. 2, which has

Table No. 2.

	$\frac{B_{w_2}}{B_{w_1}}$ or $\frac{B_{n_2}}{B_{n_1}}$	$\frac{B_{w_3}}{B_{w_0}}$ or $\frac{B_{n_3}}{B_{n_0}}$
Anticipation, or negative lag, 2 hours	1.75	8.00
Anticipation, or negative lag, 1 hour	1.61	5.50
No lag	1.48	3.70
Lag, 1 hour	1.35	2.65
Lag, 2 hours	1.22	1.90
Lag, 3 hours	1.11	1.37
Lag, 4 hours	1.00	1.00
Lag, 5 hours	.90	.73
Lag, 6 hours	.82	.52
Lag, 7 hours	.74	.38
Lag, 8 hours	.68	.27
Lag, 9 hours	.62	.19
Lag, 10 hours	.57	.12
Lag, 11 hours	.52	.08
Lag, 12 hours	.48	.05
Lag, 13 hours	.44	.03
Lag, 14 hours	.41	.01
Lag, 15 hours	.37	.00
Lag, 16 hours	.33	.00
Lag, 17 hours	.30	—

been computed directly from table No. 1 by simple division. For example, for no lag the values of B_{w_2} and B_{w_1} from table No. 1 are $\frac{20.7}{24}$ and $\frac{14.0}{24}$. Hence, in table No. 2, $\frac{B_{w_2}}{B_{w_1}}$ for no lag is shown as 1.48 $\left(=\frac{20.7}{14}\right)$.

Two values of the lag may be determined from table No. 2, one from the ratio $\frac{B_{w_2}}{B_{w_1}}$ or $\frac{B_{n_2}}{B_{n_1}}$ and the other from the ratio $\frac{B_{w_3}}{B_{w_0}}$ or $\frac{B_{n_3}}{B_{n_0}}$. The discrepancy between the two values serves as a test of the degree of accuracy with which the lag is determined by taking the mean of the two values.

VALUES OF C_w AND C_n.

In table No. 1 it may be noted that each of the following equations is either exactly or nearly true:

$$B_{w_0}+B_{w_2}=C_w \tag{36}$$

$$B_{w_1}+B_{w_3}=C_w \tag{37}$$

$$B_{n_0}+B_{n_2}=C_n \tag{38}$$

$$B_{n_1}+B_{n_3}=C_n \tag{39}$$

If table No. 2 shows equations (36) to (39) to be true for the particular case under consideration, then C_w and C_n may be computed most conveniently directly from these equations, regardless of the lag.

If the particular case is for no lag, for example, the table shows that two values of C_w may be computed, after said lag is known, from the two equations, written from table No. 1:

$$B_{w_0}+B_{w_2}=\frac{23.4}{24}C_w=0.975C_w \text{ and } B_{w_1}+B_{w_3}=C_w$$

For the cases actually encountered in the investigation, equations (36) to (39) were used as being sufficiently accurate.

From these equations (36) to (39) two values of C_w and two values of C_n may be determined from each least-square solution. The discrepancy between the two values serves as a test of the degree of accuracy with which C_w or C_n is determined by taking the mean of the two values.

ASSUMED UNIFORM CHANGE IN BAROMETRIC GRADIENTS.

The derivation of the form of observation equations shown as equation (30) on page 20 is based on an assumption which is stated explicitly below, for convenient reference, as assumption No. 3.

Assumption No. 3.

It is assumed that the barometric gradients along parallels and along meridians vary at a uniform rate in each 12-hour interval between the epochs 8 a.m. and 8 p.m. for which the forecast maps show the facts.

Assumption No. 3 is merely an approximation adopted to simplify the computation. The rate at which any barometric gradient changes varies continuously in general. The errors in the final computed results introduced by assumption No. 3 are believed to be small. Such errors are discussed later in an appropriate context.

The preceding statement in regard to the derivation of the form of the observation equations for the least-square solutions which determine the barometric constants is written primarily with reference to Lake Erie. The only modification necessary to make the statement appropriate for Lake Michigan–Huron is that indicated on pages 15–16, where it is shown that for Lake Michigan–Huron the barometric gradients are to be taken from expressions (14) and (15) rather than from expressions (12) and (13), which were used on Lake Erie. The corresponding changes must be made in equations (16) and (17). For the locations of the points 3, 4, 5, and 6 used in expressions (14) and (15), see page 15 and plate 2.

EXAMPLE OF OBSERVATION EQUATIONS FOR BAROMETRIC EFFECT.

The form of the observation equations for a least-square solution to determine barometric effects is that shown in equation (30), page 20, which is here repeated for convenience as equation (40):

$$b_{w0}B_{w0}+b_{w1}B_{w1}+b_{w2}B_{w2}+b_{w3}B_{w3}+b_{n0}B_{n0}+b_{n1}B_{n1}+b_{n2}B_{n2}+b_{n3}B_{n3}+I=V \quad (40)$$

The meanings of the separate terms in the equation are also repeated here for convenience.

The current day is defined as the day to which the equation is assigned in listing the equations.

For Lake Erie:

b_{w0} = (6–8) at 8 p.m. on the day before the day preceding the current day *minus* (6–8) at 8 a.m. on that preceding day.

b_{w1} = (6–8) at 8 a.m. of the preceding day *minus* (6–8) at 8 p.m. of that day.

b_{w2} = (6–8) at 8 p.m. of the preceding day *minus* (6–8) at 8 a.m. of the current day.

b_{w3} = (6–8) at 8 a.m. of the current day *minus* (6–8) at 8 p.m. of the current day.

The quantity (6–8) is the barometric pressure at the point marked 6 on plate 2 minus the barometric pressure at the point marked 8 on that plate. The barometric pressure is expressed in inches of mercury.

For Lake Michigan–Huron, use (4–5) instead of (6–8); see plate 2.

The appropriate modification for any other lake is evident. A point to the westward of the lake is to be used in the place of the point 6, and one to the eastward and in the same parallel is to be used in the place of point 8.

For Lake Erie:

b_{n_0} = (5–7) at 8 p.m. on the day before the day preceding the current day *minus* (5–7) at 8 a.m. on that preceding day.

b_{n_1} = (5–7) at 8 a.m. of the preceding day *minus* (5–7) at 8 p.m. of that day.

b_{n_2} = (5–7) at 8 p.m. of the preceding day *minus* (5–7) at 8 a.m. of the current day.

b_{n_3} = (5–7) at 8 a.m. of the current day *minus* (5–7) at 8 p.m. of the current day.

The quantity (5–7) is the barometric pressure at the point marked 5 on plate 2 minus the barometric pressure at the point marked 7 on that plate.

For Lake Michigan–Huron, use (3–6) instead of (5–7); see plate 2.

The appropriate modification for any other lake is to use in place of point 5 (for Lake Erie) a point to the northward of the lake, and a point to the southward of the lake and in the same meridian in the place of point 7.

B_{w_0}, B_{w_1}, B_{w_2}, B_{w_3}, B_{n_0}, B_{n_1}, B_{n_2}, and B_{n_3} are constants of which the values are to be determined from the least-square solution.

I is the observed rise in the water surface, at the gage station from the preceding day to the current day, corrected for wind effects if such corrections are available, and corrected for inflow from the next lake above, outflow to the next lake below, and rainfall on the lake surface. In other words, ignoring the various corrections for the moment, I is the mean elevation of the water surface on the current day minus the mean elevation of the water surface on the preceding day.

The residual v in the second member of equation (40) is to be derived by substitution of the computed values of B_{w_1}, B_{w_2}, . . . B_{n_2}, B_{n_3} in the observation equations after the solution is complete. The residual v, one for any observation equation, is the discrepancy between the theory and the observed facts for the particular day.

The following set of observation equations for Milwaukee for the month of September 1910 serve as a typical illustration. They are a part of the equations for solution K_2, which included in all 186 such equations covering the months June to September 1910 and June to September 1911—8 months in all.

No record of the elevation of the water surface was obtained from the gage at Milwaukee on September 2, 3, and 4.

The following possible equations were rejected: for September 2–5 (as one equation), for September 7, for September 8, for September 17, for September 24, for September 25, for September 26, and for September 28. It is unusual for so many rejections to occur in any one month. As shown, the equations for certain days were combined in pairs to make one equation in each case, namely, for August 31 and September 1 and for September 12 and 13. Each combined equation was obtained by adding corresponding terms of the separate equations.

OBSERVATION EQUATIONS.

Date 1910 Sept.	
(Aug.) 31–1....	$+5B_{w_0}-28B_{w_1}-12B_{w_2}+9B_{w_3}+23B_{n_0}+6B_{n_1}-23B_{n_2}+3B_{n_3}-9=v_1$
6....	$+2B_{w_0}-3B_{w_1}-9B_{w_2}-15B_{w_3}-6B_{n_0}+6B_{n_1}+27B_{n_2}-7B_{n_3}-213=v_2$
9....	$+3B_{w_0}-36B_{w_1}-6B_{w_2}+20B_{w_3}+5B_{n_0}-19B_{n_1}+1B_{n_2}+9B_{n_3}+21=v_3$
10....	$-6B_{w_0}+20B_{w_1}+11B_{w_2}+9B_{w_3}+1B_{n_0}+9B_{n_1}+4B_{n_2}-8B_{n_3}+7=v_4$
11....	$+11B_{w_0}+9B_{w_1}+3B_{w_2}-3B_{w_3}+4B_{n_0}-8B_{n_1}+4B_{n_2}-14B_{n_3}+45=v_5$
12–13....	$-22B_{w_0}-6B_{w_1}-22B_{w_2}+1B_{w_3}-9B_{n_0}-12B_{n_1}-6B_{n_2}+11B_{n_3}+44=v_6$
14....	$+3B_{w_0}+4B_{w_1}-1B_{w_2}+5B_{w_3}+7B_{n_0}+9B_{n_1}+10B_{n_2}-4B_{n_3}-112=v_7$
15....	$-1B_{w_0}+5B_{w_1}+3B_{w_2}+7B_{w_3}+10B_{n_0}-4B_{n_1}+3B_{n_2}+0B_{n_3}+68=v_8$
16....	$+3B_{w_0}+7B_{w_1}-2B_{w_2}+10B_{w_3}+3B_{n_0}+0B_{n_1}+3B_{n_2}-3B_{n_3}-3=v_9$
18....	$+5B_{w_0}-10B_{w_1}-21B_{w_2}+19B_{w_3}+16B_{n_0}-20B_{n_1}-31B_{n_2}+10B_{n_3}+277=v_{10}$
19....	$-21B_{w_0}+19B_{w_1}-3B_{w_2}-3B_{w_3}-31B_{n_0}+10B_{n_1}+5B_{n_2}+8B_{n_3}-35=v_{11}$
20....	$-3B_{w_0}-3B_{w_1}-9B_{w_2}-4B_{w_3}+5B_{n_0}+8B_{n_1}+11B_{n_2}-9B_{n_3}-163=v_{12}$
21....	$-9B_{w_0}-4B_{w_1}-1B_{w_2}+9B_{w_3}+11B_{n_0}-9B_{n_1}-14B_{n_2}+4B_{n_3}+118=v_{13}$
22....	$-1B_{w_0}+9B_{w_1}+16B_{w_2}+6B_{w_3}-14B_{n_0}+4B_{n_1}+5B_{n_2}+8B_{n_3}+68=v_{14}$
23....	$+16B_{w_0}+6B_{w_1}-18B_{w_2}-2B_{w_3}+5B_{n_0}+8B_{n_1}-16B_{n_2}-12B_{n_3}+110=v_{15}$
27....	$+13B_{w_0}+13B_{w_1}-20B_{w_2}-15B_{w_3}-4B_{n_0}-2B_{n_1}+27B_{n_2}-5B_{n_3}-92=v_{16}$
29....	$+7B_{w_0}+10B_{w_1}+7B_{w_2}+2B_{w_3}+3B_{n_0}-12B_{n_1}+9B_{n_2}+1B_{n_3}+108=v_{17}$
30....	$+7B_{w_0}+2B_{w_1}+6B_{w_2}-10B_{w_3}+9B_{n_0}+1B_{n_1}+12B_{n_2}-1B_{n_3}-113=v_{18}$

The basis on which rejections and combinations were made is shown later in connection with the discussion of the accuracy of the computed barometric effects.

BAROMETRIC TERMS IN OBSERVATION EQUATIONS.

For Lake Michigan the barometric pressures used in the computations were those read at points 3, 4, 5, and 6 as shown on plate 2. The values of the barometric pressures at these points at 8 a.m. and 8 p.m. (75th meridian time) of each day for a part of September 1910 are shown in table No. 3. The table also shows the values of the differences of barometric pressure (4–5) and (3–6) which were used in the preparation of a part of the above observation equations.

In the above equations the values of b_{w_1}, b_{w_2}, . . . b_{n_2} and b_{n_3} are expressed in units of 0.01 inch, and the absolute term I, the rise of the water surface, is expressed in units of 0.001 foot. This arbitrary selection of units was made in order that the average magnitude of quantities in the various columns, as the equations are arranged in the example given, might be about the same. To have them about the same increases the accuracy of certain checks against error in the computations.

The derivation of the values of b_{w_1}, b_{w_2}, . . . b_{n_2} and b_{n_3} as given in the observation equations may be verified from the table for the period which it covers. For example, note that the exceptionally large value of b_{w_1} in the equation for September 9 is (4–5) at 8 a.m. of September 8, (-17), minus (4–5) at 8 p.m. of September 8, $(+19)$, or -36. This is the most rapid change of barometric difference shown anywhere in table No. 3. Note that

there was also a rather rapid change in the barometric difference in the meridian, difference 3–6, between 8 a.m. and 8 p.m. on September 8 from which the term b_{n1} in the September 9 observation equation is derived.

TABLE No. 3.

Date 1910.	Hour.	Barometric pressure in inches of mercury.				In units of 0.01 inch.	
		At point 3.	At point 4.	At point 5.	At point 6.	Diff. (4–5)	Diff. (3–6)
Sept. 1...........	8 a.m.	30.32	30.30	30.19	30.20	+11	+12
1...........	8 p.m.	30.23	30.18	30.16	30.14	+ 2	+ 9
2...........	8 a.m.	30.19	30.13	30.23	30.16	−10	+ 3
2...........	8 p.m.	29.83	29.80	29.96	29.87	−16	− 4
3...........	8 a.m.	29.70	29.76	29.73	29.77	+ 3	− 7
3...........	8 p.m.	29.85	29.82	29.74	29.80	+ 8	+ 5
4...........	8 a.m.	29.90	29.85	29.95	29.85	−10	+ 5
4...........	8 p.m.	29.93	29.83	29.95	29.84	−12	+ 9
5...........	8 a.m.	29.92	29.77	29.91	29.77	−14	+15
5...........	8 p.m.	29.89	29.78	29.89	29.80	−11	+ 9
6...........	8 a.m.	29.70	29.78	29.80	29.88	− 2	−18
6...........	8 p.m.	29.93	30.03	29.90	30.04	+13	−11
7...........	8 a.m.	30.19	30.15	30.13	30.23	+ 2	− 4
7...........	8 p.m.	30.02	30.04	30.18	30.16	−14	−14
8...........	8 a.m.	29.90	29.94	30.11	30.09	−17	−19
8...........	8 p.m.	29.99	30.08	29.89	29.99	+19	0
9...........	8 a.m.	30.20	30.28	30.03	30.21	+25	− 1
9...........	8 p.m.	30.13	30.18	30.13	30.23	+ 5	−10
10...........	8 a.m.	30.12	30.18	30.24	30.26	− 6	−14
10...........	8 p.m.	30.03	29.99	30.14	30.09	−15	− 6
11...........	8 a.m.	30.02	30.02	30.20	30.12	−18	−10
11...........	8 p.m.	30.03	29.25	30.10	29.99	−15	+ 4
12...........	8 a.m.	30.21	30.22	30.12	30.04	+10	+17
12...........	8 p.m.	30.37	30.34	30.21	30.22	+13	+15
13...........	8 a.m.	30.45	30.44	30.34	30.37	+10	+ 8
13...........	8 p.m.	30.31	30.36	30.30	30.32	+ 6	− 1
14...........	8 a.m.	30.30	30.40	30.33	30.41	+ 7	−11
14...........	8 p.m.	30.28	30.31	30.29	30.35	+ 2	− 7
15...........	8 a.m.	30.30	30.35	30.36	30.40	− 1	−10
15...........	8 p.m.	30.21	30.26	30.34	30.31	− 8	−10

Table No. 4 shows the observed elevations referred to mean sea-level of the water surface at the Milwaukee gage, the observed rise of water surface for each day, the corrections to observed rise for known inflow, outflow, and rainfall on the lake surface, the corrections to the observed rise for wind effects, and the corrected rise as used in the observation equations as the I term.

CHANGE OF ELEVATION USED IN OBSERVATION EQUATIONS.

Each observed daily elevation is the mean of 24 hourly elevations obtained from the gage record.

The inflow to Lake Michigan–Huron, through the St. Marys River, from Lake Superior was determined each day by the U. S. Lake Survey by means of a recording gage giving the elevation of the water surface at Sault Ste. Marie above the rapids. Similarly, the outflow from Lake Michigan–Huron to Lake Erie was determined for each day from the recorded elevation of the water surface at Fort Gratiot and at St. Clair Flats in the St. Clair River. In each of these cases the recorded elevations of water surface were converted to volume of stream-flow by means of the known relations between these quantities which had been established by river gagings made by the U. S. Lake Survey. As these corrections for inflow and outflow are small and are abundantly accurate for the purpose, the explanation in detail of their computation is not here given.

No correction was applied for run-off into Lake Michigan–Huron from the surrounding land-drainage area. This quantity was very difficult to determine at the time this investigation was made. So, too, no correction is applied for evaporation, for a similar reason.

The amount of rise of water surface of the lake on each day, produced by rainfall on the lake surface, was computed from a number of rain gages operated by the U. S. Weather Bureau and the Weather Service of Canada.

For convenience the three corrections—for inflow, for outflow, and for rainfall on the lake surface—were grouped together in the computation as shown in the third column of table No. 4. These three corrections are given separately in the last three columns of the table.

Note that the difference between outflow and inflow, or what might be called net outflow, is no more than 0.008 foot on any day in September 1910. Note that the maximum rise or fall during any day of the month due to combined inflow, outflow, and rainfall on the lake surface was only 0.033 foot. It is clear, after comparing these values with the observed rise of the lake surface at Milwaukee for each day, as shown in table No. 4, that the observed rise is evidently due to causes other than these three, which are minor in their effect upon the daily fluctuations of level as compared with other causes which are operating.

The corrections for wind effects as shown, at a maximum only 0.001 foot for September 24 and September 25, were computed by methods shown in detail later in this publication.

In all columns except the second the change shown is for that from the preceding day to the day indicated in the first column on that line. For example, −0.180 foot is the rise of water surface from September 5 to September 6, namely, 580.07–580.25. Similarly, −0.033 foot is the correction for rainfall, inflow, and outflow combined from September 5 to September 6.

The observation equations were arranged in groups of one month each.

The groups were studied separately as well as in combination with each other as a single set of equations for one complete solution.

TABLE No. 4—*For Milwaukee, Unit 0.001 foot.*

	Observed elevation of water surface.	Rise of water surface.	Correction for rainfall, inflow and outflow.	Correction for wind effect.	Corrected rise.	Correction for rainfall.	Correction for inflow.	Correction for outflow.
1910	*feet.*							
Sept. 1...	580.32	....	+ 7	0	....	− 1	−5	+13
2...	....	....	+ 7	0	....	0	−5	+12
3...	....	....	+ 1	0	....	− 7	−5	+13
4...	....	....	− 2	0	....	−10	−5	+13
5...	580.25	− 70	−26	0	....	−33	−5	+12
6...	.07	−180	−33	0	−213	−40	−5	+12
7...	.00	− 70	− 2	0	− 72	−10	−5	+13
8...	.23	+230	+ 2	0	+232	− 6	−5	+13
9...	.25	+ 20	+ 1	0	+ 21	− 7	−5	+13
10...	.25	0	+ 7	0	+ 7	− 1	−5	+13
11...	.29	+ 40	+ 5	0	+ 45	− 2	−5	+12
12...	.51	+220	−19	0	+201	−27	−5	+13
13...	.37	−140	−17	0	−157	−25	−5	+13
14...	.25	−120	+ 8	0	−112	0	−5	+13
15...	.31	+ 60	+ 8	0	+ 68	0	−5	+13
16...	.30	− 10	+ 7	0	− 3	0	−5	+12
17...	.16	−140	− 2	0	−142	− 9	−5	+12
18...	.44	+280	− 3	0	+277	−11	−5	+13
19...	.40	− 40	+ 5	0	− 35	− 2	−5	+12
20...	.23	−170	+ 7	0	−163	0	−5	+12
21...	.34	+110	+ 8	0	+118	0	−5	+13
22...	.40	+ 60	+ 8	0	+ 68	0	−4	+12
23...	.51	+110	0	0	+110	− 7	−5	+12
24...	.71	+200	−20	−1	+179	−27	−4	+11
25...	.46	−250	−15	+1	−264	−22	−5	+12
26...	.37	− 90	+ 3	0	− 87	− 5	−4	+12
27...	.28	− 90	− 2	0	− 92	− 9	−5	+12
28...	.14	−140	+ 3	0	−137	− 5	−5	+13
29...	.24	+100	+ 8	0	+108	0	−4	+12
30...	.12	−120	+ 7	0	−113	0	−5	+12

EXAMPLE OF NORMAL EQUATIONS FOR BAROMETRIC EFFECTS.

In each solution a set of normal equations was first formed for each month, in the usual way, from the observation equations for that month. These sets of normal equations were then available for separate study for each month.

These monthly normal equations were later combined by addition, term by term, to form the final set of normal equations for the solution.

The final normal equations for solution K_2 are given below as table No. 5. Solution K_2 served to give the final determination of the barometric effects at Milwaukee.

TABLE No. 5—*Normal Equations, Solution* K_2, *Milwaukee.*

$$
\begin{aligned}
+17944B_{w0}-2552B_{w1}-3955B_{w2}-2447B_{w3}+4802B_{n0}-573B_{n1}+2817B_{n2}+365B_{n3}-17427&=0\\
-2552B_{w0}+16336B_{w1}+3358B_{w2}-4622B_{w3}-3304B_{n0}+4130B_{n1}+2024B_{n2}-1482B_{n3}+41911&=0\\
-3955B_{w0}+3358B_{w1}+22035B_{w2}-3147B_{w3}-4916B_{n0}-1181B_{n1}+4269B_{n2}+2774B_{n3}+53880&=0\\
-2447B_{w0}-4622B_{w1}-3147B_{w2}+15104B_{w3}-347B_{n0}-1438B_{n1}-4245B_{n2}+2904B_{n3}+11035&=0\\
+4802B_{w0}-3304B_{w1}-4916B_{w2}-347B_{w3}+13444B_{n0}-1437B_{n1}-3300B_{n2}-4065B_{n3}-8746&=0\\
-573B_{w0}+4130B_{w1}-1181B_{w2}-1438B_{w3}-1437B_{n0}+12149B_{n1}-457B_{n2}-65B_{n3}-61166&=0\\
+2817B_{w0}+2024B_{w1}+4269B_{w2}-4245B_{w3}-3300B_{n0}-457B_{n1}+17206B_{n2}-1586B_{n3}-73936&=0\\
+365B_{w0}-1482B_{w1}+2774B_{w2}+2904B_{w3}-4065B_{n0}-65B_{n1}-1586B_{n2}+10168B_{n3}+2900&=0
\end{aligned}
$$

These normal equations depend upon observations extending over 220 days, which were expressed in 186 observation equations. Some of the observation equations covered two or more days each.

The solution of these normal equations gave the following values for the unknowns, expressed in the units used in the observation equations:

$$
\begin{array}{ll}
B_{w0}=-1.78 & B_{n0}=+1.99\\
B_{w1}=-4.34 & B_{n1}=+6.53\\
B_{w2}=-2.92 & B_{n2}=+6.34\\
B_{w3}=-\ .90 & B_{n3}=+2.03
\end{array}
$$

EXAMPLE OF SUBSTITUTION IN OBSERVATION EQUATIONS FOR BAROMETRIC EFFECTS.

The above values by substitution in the observation equations for September 1910 as given on page 26 gave the equations which served to determine the residuals v for this particular month. From such residuals the probable errors were computed. These residuals are the discrepancies between the theory on which the observation equations were based, on the one hand, and the observed facts, on the other hand.

In the following tabular arrangement of the substitution in observation equations the heading on each column identifies the term of the observation equations as shown on page 26. As a specific example, note that the value given in the B_{w1} column for September 9, namely, $+156$, is the product of the quantity -36 in the September 9 observation equation, which is the coefficient of B_{w1} times the final value of B_{w1}, namely, -4.34.

The values of v, the residuals, were obtained by adding all terms in the first member of the equation. If the agreement between theory and observation were perfect, all v's would be zero.

Note that there were 8 days in September 1910 when the observed rise (or fall) at Milwaukee, corrected for rainfall, inflow, outflow, and wind, was more than 0.1 foot (corresponding to 100 in the I column in the following substitution in observation equations), namely, September 6, 14, 18, 20, 21,

23, 29, and 30. The largest residual v for any one of these 8 days is $+0.079$ foot on September 18. For that day the I term or observed rise (with the small noted corrections) as expressed by the I term was $+0.277$ foot. In other words, of the unusually large observed rise of 0.280 foot (see table No. 4) from September 17 to September 18, 0.003 foot is accounted for by rainfall, inflow, and outflow, 0.198 foot is accounted for as barometric effects, and only 0.079 foot, or only 28 per cent of the observed rise, is left unaccounted for.

Substitution in Observation Equations for September 1910, at Milwaukee. The unit is 0.001 foot.

Date	B_{w0}	B_{w1}	B_{w2}	B_{w3}	B_{n0}	B_{n1}	B_{n2}	B_{n3}	I	v
31- 1..........	− 9	+122	+35	− 8	+46	+ 20	−127	+ 6	− 9	= +76
6..........	− 4	+ 13	+26	+13	−12	+ 39	+171	−14	−213	= +19
9..........	− 5	+156	+18	−18	+10	−124	+ 6	+18	+ 21	= +82
10..........	+11	− 87	−32	− 8	+ 2	+ 59	+ 25	−16	+ 7	= −39
11..........	−20	− 39	− 9	+ 3	+ 8	− 52	+ 25	−28	+ 45	= −67
12–13..........	− 5	+ 13	+73	− 4	+ 8	− 78	− 38	+18	+ 44	= +31
14..........	− 5	− 17	+ 3	− 4	+14	+ 59	+ 63	− 8	−112	= − 7
15..........	+ 2	− 22	− 9	− 6	+20	− 26	+ 19	0	+ 68	= +46
16..........	− 5	− 30	+ 6	− 9	+ 6	0	+ 19	− 6	− 3	= −22
18..........	− 9	+ 43	+61	−17	+32	−131	−197	+20	+277	= +79
19..........	+37	− 82	+ 9	+ 3	−62	+ 65	+ 32	+16	− 35	= −17
20..........	+ 5	+ 13	+26	+ 4	+10	+ 52	+ 70	−18	−163	= − 1
21..........	+16	+ 17	+ 3	− 8	+22	− 59	− 89	+ 8	+118	= +28
22..........	+ 2	− 39	−47	− 5	−28	+ 26	+ 32	+16	+ 68	= +25
23..........	−28	− 26	+53	+ 2	+10	+ 52	−101	−24	+110	= +48
27..........	−23	− 56	+58	+13	− 8	− 13	+171	−10	− 92	= +40
29..........	−12	− 43	−20	− 2	+ 6	− 78	+ 57	+ 2	+108	= +18
30..........	−12	− 9	−18	+ 9	+18	+ 7	+ 76	− 2	−113	= −44

THE FIVE FINAL SOLUTIONS FOR BAROMETRIC EFFECTS.

The principal facts for each of the five least-square solutions which served to give the adopted values for the quantities B_{w_0}, B_{w_1}, . . . B_{n_2}, and B_{n_3} expressing the barometric effects at the five stations, Buffalo, Cleveland, Milwaukee, Harbor Beach, and Mackinaw, are here brought together for convenient reference.

Each of the five solutions was based upon 8 months of observation of water elevations at the station named. At Buffalo and Cleveland, the months were August to October, inclusive, 1909, and June to October, inclusive, 1910. At Milwaukee, Harbor Beach, and Mackinaw, the months were June to September, inclusive, in each of the two years 1910 and 1911.

In table No. 6 the probable errors included in parentheses are estimated on the basis of those which are not so marked. These latter were computed rigorously from the residuals and the normal equations.

The values of B_{w_0}, B_{w_1}, . . . B_{n_2} and B_{n_3} as given in table No. 6 correspond to the units arbitrarily adopted for convenience in the observation equations—that is, if b_{w_0}, b_{w_1}, . . . b_{n_2} and b_{n_3} are expressed in units of 0.01 inch and the above values for B_{w_0}, B_{w_1}, . . . B_{n_2} and B_{n_3} are used, the resulting products b_{w_0} B_{w_0}, b_{w_1} B_{w_1} . . . b_{n_2} B_{n_2} and b_{n_3} B_{n_3} are barometric effects expressed in units of 0.001 foot.

TABLE No. 6—*Principal facts for each of the five final least-square solutions for barometric effects.*

Gage.	Buffalo.	Cleveland.	Milwaukee.	Harbor Beach.	Mackinaw.
Designation of solution..	L2	M2	K2	N1	O1
Days of observation used in solution...........	229	239	220	221	234
No. of observation equations................	188	213	186	202	220
B_{w0}..............	$-$ 2.22± 0.42	+0.69± 0.30	$-$1.78±(0.25)	+2.81±(0.21)	$-$ .73±(0.17)
B_{w1}..............	+ 3.85±(0.39)	+1.22±(0.30)	$-$4.34± 0.25	+5.29± 0.21	$-$ .91± 0.18
B_{w2}..............	+ 5.96±(0.37)	+1.04±(0.30)	$-$2.92±(0.23)	+5.10±(0.18)	$-$2.05±(0.17)
B_{w3}..............	+ 1.85±(0.42)	+1.30±(0.30)	$-$ .90±(0.25)	+ .68±(0.21)	$-$1.38±(0.18)
B_{n0}..............	$-$ 3.96± 0.44	+1.73± 0.30	+1.99±(0.27)	$-$1.00±(0.26)	+ .38±(0.19)
B_{n1}..............	$-$10.78±(0.45)	+4.23±(0.30)	+6.53± 0.27	$-$ .84± 0.23	$-$2.89± 0.21
B_{n2}..............	$-$11.69±(0.43)	+6.04±(0.30)	+6.34±(0.25)	+ .49±(0.22)	$-$4.70±(0.19)
B_{n3}..............	$-$ 4.82±(0.45)	+1.60±(0.30)	+2.03±(0.28)	+ .47±(0.26)	$-$1.55±(0.20)
Probable errors of a single observation of the rise of the water surface in one day	±.044 ft.	±.036 ft.	±.028 ft.	±.025 ft.	±.023 ft.

COMPUTATION OF HOURLY BAROMETRIC EFFECTS.

To secure a basis for computing the hourly barometric effects at a gage station, one must first compute the lag in each component of the barometric effect by the use of table No. 2, page 22, and compute the values of C_w and C_n by formulæ (36), (37), (38), and (39) of page 23.

At Milwaukee these computations were made from the values shown in table No. 6.

The lag in C_w was found thus:

$$\frac{B_{w2}}{B_{w1}}=\frac{-2.92}{-4.34}=+0.67 \text{ and } \frac{B_{w3}}{B_{w0}}=\frac{-0.90}{-1.78}=+0.50$$

From the first of these, according to table No. 2, the lag is 8.2 hours, and from the second it is 6.1 hours. The mean (7 hours) was adopted as the most probable value of the lag in C_w.

Similarly, the two values found for the lag in C_n were 4.3 and 4.1 hours. The adopted value was taken as 4 hours.

The values of C_w computed from formulæ (36) and (37) were, respectively, $-1.78-2.92=-4.70$ and $-4.34-0.90=-5.24$. The mean -4.97 was adopted as the most probable value of C_w.

The two values of C_n computed from formulæ (38) and (39), respectively, were $+8.33$ and $+8.56$. The mean $+8.44$ was adopted as the most probable value of C_n.

The computations of lag and of C_w and C_n were made for all five stations from the values of B_{w_0}, B_{w_1}, . . . B_{n_2}, and B_{n_3}, shown in table No. 6, with the results shown below in table No. 7.

TABLE No. 7.

	Buffalo.	Cleveland.	Milwaukee.	Harbor Beach.	Mackinaw.
Lag in C_w, hours.	− 1*	4	7	6	−2*
Lag in C_n, hours.	3	2	4	6	−1*
C_w, in same units as table No. 6	+ 4.72	+2.12	−4.97	+6.94	−2.54
C_n, in same units as table No. 6	−15.62	+6.80	+8.44	−0.44	−4.38

*Note that these three values of the lag are marked with minus signs. They are to be so used. They are anticipations rather than lags.

From these values the barometric effect at each hour at each station may be computed from the following formulæ, each of which is formula (18), page 17, modified (*a*) to adapt it to the particular lake, (*b*) to take into account the lags which are now known, and (*c*) to adapt it to the units used above in table No. 7 and in the observation equations.

In the following formulæ the quantities (6–8), (5–7), (4–5), and (3–6) are expressed in units of 0.01 inch. The computed effect, E_1, is obtained from the formulæ in units of 0.001 foot.

For Buffalo, E_1, for any hour = (6–8) (+4.72) + (5–7) (−15.62), in which (6–8) must be taken for 1 hour later and (5–7) for 3 hours earlier than the hour for which the effect is being computed. (41)

For Cleveland, E_1, for any hour = (6–8) (+2.12) + (5–7) (+6.80), in which (6–8) must be taken for 4 hours earlier and (5–7) for 2 hours earlier than the hour for which the effect is being computed. (42)

For Milwaukee, E_1, for any hour = (4–5) (−4.97) + (3–6) (+8.44), in which (4–5) must be taken for 7 hours earlier and (3–6) for 4 hours earlier than the hour for which the effect is being computed. (43)

For Harbor Beach, E_1, for any hour = (4–5) (+6.94) + (3–6) (−0.44), in which (4–5) must be taken for 6 hours earlier and (3–6) for 6 hours earlier than the hour for which the effect is being computed. (44)

For Mackinaw, E_1, for any hour = (4–5) (−2.54) + (3–6) (−4.38), in which (4–5) must be taken for 2 hours later and (3–6) for 1 hour later than the hour for which the effect is being computed. (45)

The example on page 35 shows the details of the computation of hourly barometric effects at Milwaukee on September 24, 1910. The barometric effects at Milwaukee on this day were unusually large and were changing with unusual rapidity.

It was especially desirable to secure the hourly barometric effects on September 24 with as great accuracy as possible, because this was one of the dates used in a computation of wind effects at Milwaukee. Hence, a special study of the forecast maps of this and adjacent days was made with a view to determining the time of maximum and minimum pressure at each reading point and the value of each such maximum or minimum. Within the limits shown in the example there are six points found by this special study, namely:

A maximum of 30.34 at 9 p.m. on Sept. 23 at point 3,
A maximum of 30.10 at 9 p.m. on Sept. 23 at point 6,
A maximum of 30.26 at 3 a.m. on Sept. 24 at point 5,
A minimum of 29.80 at 11 a.m. on Sept. 24 at point 6,
A minimum of 29.80 at 5 p.m. on Sept. 24 at point 4,
A minimum of 29.79 at 11 p.m. on Sept. 24 at point 3.

Each of these maxima and minima is inclosed in a parenthesis in the example of computation given.

Consult plate 2 for the location of the points 3, 4, 5, and 6 for which the barometric pressures were read from the forecast maps.

Opposite each maximum or minimum the pressure as given in square brackets for the companion point was obtained by interpolation between the next preceding and next following value at that point, on the assumption that during the interval the pressure changed at a uniform rate.

The values in the columns (3–6) and (4–5) which are not included in square brackets were obtained directly by subtraction from the preceding

columns. The values in these columns which are inclosed in square brackets were obtained by interpolation from the unbracketed values in the same columns, on the assumption that the rate of change of the barometric gradient was constant during each interval between unbracketed values.

Example of computation of hourly barometric effects—Milwaukee.

Date and hour.	Barometric pressure, at point.						N–S effect.	E–W effect.	E_1 total effect.
	3.	6.	4.	5.	(3–6).	(4–5).			
	inches.	*inches.*	*inches.*	*inches.*	*inches.*	*inches.*	*feet.*	*feet.*	*feet.*
Sept. 23.									
8 p.m......	30.34	30.09	30.20	30.20	+.25	+.00	....	....	....
9	(30.34)	(30.10)	....	....	+.24	[−.02]	....	....	....
10	....	....	....	....	[+.25]	[−.05]	....	....	....
11 p.m......	....	....	....	....	[+.27]	[−.08]	....	....	....
M..........	....	....	....	....	[+.28]	[−.10]	....	....	....
Sept. 24									
1 a.m......	....	....	....	....	[+.30]	[−.12]	+.20	.00	+.20
2	....	....	....	....	[+.31]	[−.16]	+.21	.00	+.21
3	....	....	[30.08]	(30.26)	[+.33]	−.18	+.23	.00	+.23
4	....	....	....	....	[+.34]	[−.19]	+.24	+.01	+.25
5	....	....	....	....	[+.36]	[−.20]	+.25	+.02	+.27
6	....	....	....	....	[+.37]	[−.21]	+.26	+.04	+.30
7	....	....	....	....	[+.38]	[−.22]	+.28	+.05	+.33
8	30.22	29.82	29.99	30.22	+.40	−.23	+.29	+.06	+.35
9	....	....	....	....	[+.38]	[−.23]	+.30	+.08	+.38
10		....	....	....	[+.35]	[−.22]	+.31	+.09	+.40
11 a.m......	[30.13]	(20.80)	....	....	+.33	[−.22]	+.32	+.09	+.41
N..........	....	....	....	....	[+.29]	[−.21]	+.34	+.10	+.44
1 p.m......	....	....	....	....	[+.24]	[−.21]	+.32	+.10	+.42
2	....	....	....	....	[+.20]	[−.20]	+.30	+.11	+.41
3	....	....	....	....	[+.16]	[−.20]	+.28	+.11	+.39
4	....	....	....	....	[+.11]	[−.19]	+.24	+.11	+.35
5	....	....	(29.80)	[29.99]	[+.07]	−.19	+.20	+.11	+.31
6	....	....	....	....	[+.03]	[−.16]	+.17	+.11	+.28
7	....	....	....	....	[−.02]	[−.12]	+.14	+.10	+.24
8	29.87	29.93	29.83	29.91	−.06	−.08	+.09	+.10	+.19
9	....	....	....	....	[−.10]	[−.07]	+.06	+.10	+.16
10	....		....	....	[−.14]	[−.06]	+.03	+.10	+.13
11 p.m......	(29.79)	[29.96]	....	....	−.17	[−.05]	−.01	+.09	+.08
M.........	....	....	....	....	[−.17]	[−.04]	−.05	+.09	+.04
								Mean.	+.282

The insertion of the six maximum and minimum points, and of the corresponding unbracketed values in the (3–6) and (4–5) columns, probably gave a slightly increased accuracy to the computation for this particular day. These maximum and minimum points can be determined with difficulty and with but a small degree of accuracy from the forecast maps by studying the rate and direction of progress of the isobars in each 12-hour interval. Usually, the maxima and minima are poorly defined and differ but little from the values which would be obtained by direct interpolation between the preceding and following 8 o'clock values. For these reasons the hourly

barometric effects have been computed as a rule without the insertion of maximum and minimum points, in the belief that the possible gain in accuracy thereby sacrificed is negligible.

The separate values of N–S effect and of E–W effect, and their sum E_1, the total barometric effect, were computed from formula (43). Note that the computed maximum barometric effect occurred at noon on September 24 corresponding to the value +40 in the column headed (3–6).

To obtain hourly elevations of the mean surface of Lake Michigan–Huron from observed hourly elevations of the water surface at Milwaukee, one must subtract the barometric effect for each hour computed as shown in the example.

COMPUTATION OF DAILY BAROMETRIC EFFECTS.

The mean of the 24 hourly barometric effects for any day is the best value obtainable for the daily barometric effect.

To obtain daily elevations of the mean surface of Lake Michigan–Huron, one must subtract the daily barometric effect for each day from the mean of the 24 hourly elevations of the water surface at Milwaukee for that day.

To compute the barometric effect for each day by first computing the 24 hourly barometric effects for that day and then taking the mean is an unnecessarily slow and laborious procedure. The method of computation indicated below is much more rapid and is believed to be but slightly less accurate.

In the formula for the observation equations as shown in (40), page 24, the first eight terms $b_{w0}\ B_{w0}$, $b_{w1}\ B_{w1}\ .\ .\ .\ b_{n2}\ B_{n2}$, $b_{n3}\ B_{n3}$ express the change in barometric effect from one day to the next. The quantities B_{w0}, B_{w1}, . . . B_{n2}, B_{n3} have been determined by the least-square solutions, and their values are shown, for each of five stations, in table No. 6, page 32. Therefore, the following formulæ express the change in the barometric effect from one day to the next at the stations named:

At Buffalo

$$-2.22b_{w0}+3.85b_{w1}+5.96b_{w2}+1.85b_{w3}-3.96b_{n0} -10.78b_{n1}-11.69b_{n2}-4.82b_{n3} \qquad (46)$$

At Cleveland

$$+0.69b_{w0}+1.22b_{w1}+1.04b_{w2}+1.30b_{w3}+1.73b_{n0} +4.23b_{n1}+6.04b_{n2}+1.60b_{n3} \qquad (47)$$

At Milwaukee

$$-1.78b_{w0}-4.34b_{w1}-2.92b_{w2}-.90b_{w3}+1.99b_{n0} +6.53b_{n1}+6.34b_{n2}+2.03b_{n3} \qquad (48)$$

At Harbor Beach

$$+2.81b_{w0}+5.29b_{w1}+5.10b_{w2}+.68b_{w3}-1.00b_{n0} -.84b_{n1}+.49b_{n2}+.47b_{n3} \qquad (49)$$

At Mackinaw

$$-0.73b_{w0}-.91b_{w1}-2.05b_{w2}-1.38b_{w3}+.38b_{n0} -2.89b_{n1}-4.70b_{n2}-1.55b_{n3} \qquad (50)$$

For the definitions of the meanings of b_{w0}, b_{w1} . . . b_{n2}, b_{n3}, consult page 24. In the above formulæ the unit in which these quantities are to be expressed is 0.01 inch. In these formulæ the unit in which the barometric effect is expressed is in units of 0.001 foot.

The method of computation actually used to secure the barometric effects for each day consists of five steps, as follows:

(1) The hourly barometric effects were computed by the method set forth on pages 32–36 for the first, the last, and a few intermediate days of the long series of days under consideration. The daily barometric effect was obtained for each of these selected days by taking the mean of the 24 hourly effects for that day—the most accurate method.

(2) The change in barometric effect from day to day throughout the whole series was computed from the appropriate one of formulæ (46) to (50).

(3) The changes from day to day computed in step (2) were applied one by one to the computed barometric effect for the first day of the series obtained in step (1) to secure the barometric effect for each day in turn up to and including the next selected day on which the daily barometric effect had been computed in step (1).

(4) At this point, the second selected day, a discrepancy appeared between the barometric effect as computed for this day by step (1) and as computed by step (3). This discrepancy was distributed proportionally to time between the first and the second selected day, and all intermediate values corrected, so as to make the discrepancy disappear at the second selected day. The values so corrected were adopted as sufficiently accurate.

(5) Steps (3) and (4) were taken from the second to the third selected day, from the third to the fourth selected day, and so on to the end of the series.

The computation for August 26 to September 24, 1910, at Milwaukee, made in accordance with the above statement, is shown on the following page. The unit is one foot.

The values in the column headed "Change in barometric correction" were computed from formula (48). Each value is the change in barometric correction from the day before to the day indicated on the line on which the value is placed.

Of the values in the column headed "Barometric correction based directly on computed values," those included in parentheses for the selected days were computed from hourly barometric effects as indicated in step (1), and the others, without parentheses, were computed as indicated in step (3) by applying from day to day the changes shown in the preceding column.

At the second selected day, September 4, there are two values in the third column, namely, +0.045, computed by step (3), and (−0.070), computed by step (1). The discrepancy is −0.115 foot. This discrepancy was distributed over the interval August 26–September 4 at $\frac{0.115}{9} = 0.0128$ foot per day, as shown in the fourth column.

Example of computation of daily barometric effects (Milwaukee, Aug. 26 to Sept. 24, 1910).

Date.	Barometric correction.				Elevation of water surface.	
	Change in correction.	Based directly on computed values.	To conform to values computed by step (1).	Final.	Observed.	Corrected.
Aug. 26...	....	(+.238)	.000	+.238	580.00	580.238
27...	−.018	+.220	−.013	+.207	.14	.347
28...	−.176	+.044	−.026	+.018	.33	.348
29...	−.098	−.054	−.038	−.092	.34	.248
30...	+.058	+.004	−.051	−.047	.34	.293
31...	+.059	+.063	−.064	−.001	.24	.239
Sept. 1...	−.015	+.048	−.077	−.029	.32	.291
2...	+.011	+.059	−.090	−.031	....	....
3...	+.065	+.124	−.102	+.022	....	....
4...	−.079	+.045 (−.070)	.000	−.070	....	....
5...	−.096	−.166	.000	−.166	580.25	580.084
6...	+.232	+.066	+.001	+.067	.07	.137
7...	+.019	+.085	+.002	+.087	.00	.087
8...	−.022	+.063	+.002	+.065	.23	.295
9...	+.061	+.124	+.002	+.126	.25	.376
10...	−.046	+.078	+.003	+.081	.25	.331
11...	−.112	−.034	+.004	−.030	.29	.260
12...	−.077	−.111	+.004	−.107	.51	.403
13...	+.093	−.018	+.004	−.014	.37	.356
14...	+.105	+.087 (+.092)	.000	+.092	.25	.342
15...	−.022	+.070	.000	+.070	.31	.380
16...	−.019	+.051	+.001	+.052	.30	.352
17...	+.001	+.052	+.001	+.053	.16	.213
18...	−.198	−.146	+.002	−.144	.44	.296
19...	+.018	−.128	+.002	−.126	.40	.274
20...	+.162	+.034	+.002	+.036	.23	.266
21...	−.090	−.056	+.003	−.053	.34	.287
22...	−.043	−.099	+.003	−.096	.40	.304
23...	−.062	−.161	+.004	−.157	.51	.353
24...	−.125	−.286 (−.282)	.000	−.282	.71	.428

The corrections shown in the fourth column were applied to the values shown in the third column to obtain the final adopted values of the barometric corrections as shown in the fifth column.

The rule ordinarily observed in regard to selected days was to place them not more than 1 month apart in any case and not more than 10 days apart over any interval through which the discrepancy to be distributed exceeded 0.070 foot.

The discrepancy in question is due to two causes: (*a*) to omitted decimal places in the computations, and (*b*) to the fact that the process of computing

the hourly barometric effects, and thence the daily barometric effects, by step (1) involves a smoothing out of the discrepancies between the values of B_{w0}, B_{w1}, . . . B_{n2}, B_{n3} as derived from the least-square solution, and therefore does not agree exactly with the computations made from formulæ (46) to (50). The rule stated in the preceding paragraph was adopted, as a result of experience, as probably giving the best balance between extreme accuracy, on the one hand, and large expenditure of time in computation, on the other hand.

THEORETICAL BASIS FOR WIND OBSERVATION EQUATIONS.

The formula for wind effect which has been adopted as the basis for the observation equations is as follows:

$$W = C_x\left(\frac{h^{2\cdot 4}}{100}\right)\Sigma_x \tag{51}$$

W is the effect, at a given time, of the wind, at a given gage station, in elevating the water surface at that station above the mean elevation of the whole surface of the lake. h is the velocity of the wind at that station. Σ_x is a quantity appropriate to the station, for each wind direction, which expresses the relation between the effect of a wind of a certain velocity, on the one hand, and the depth of the lake at every point, the shape of the bottom, the horizontal dimensions of the lake, and the shape of its shore, on the other hand.

A wind blowing across a lake drives the surface water to leeward at every point on the surface at a rate dependent upon the wind velocity. This surface water delivered toward the lee shore tends to raise the surface elevation of that part of the lake. As soon as this action has established a surface slope downward to windward, gravity tends to set up a current to windward, which current extends to the full depth of the lake, from the surface to the bottom. Said return current to windward is a function of the surface slope, tending to be greater the greater the slope. When a steady régime has been established for a wind of a certain direction and velocity, the total volume of water per unit of time delivered to windward across any line which may be drawn completely across the lake is necessarily equal to the total volume of surface water delivered per unit of time to leeward across that line. If it were otherwise, the surface elevations at some parts of the lake would necessarily be changing and the régime would not be steady.

Formula (51) expresses the wind effect after the steady régime has been established for a wind of any given direction and velocity.

The explanation of the theoretical basis of formula (51) is given in the following successive steps:

(1) For an assumed wind of constant direction and constant velocity, and for a narrow strip of the lake parallel to the direction of the wind, the relation at each point of the strip, during a steady régime, between the

depth of the water and its surface slope is established. In this part of the exposition, each of the various strips, each parallel to the wind direction, across the lake from windward to leeward is assumed to act independently of every other such strip.

(2) It is recognized that the various strips across a lake do not act independently. The method of applying the relation derived in (1) to an actual lake with its irregular bottom and shores is set forth.

(3) A statement is made in regard to the manner in which the exponent of h (2.4) has been derived.

RELATION BETWEEN DEPTH AND SLOPE PRODUCED BY WIND.

The Chezy formula for the flow of water in an open channel is the standard formula usually given in the text-books on hydraulics. It is the fundamental basis from which other more elaborate formulæ have been built up.

The Chezy formula is

$$V = C(RS)^{\frac{1}{2}} \tag{52}$$

In this formula,

V = the mean velocity of flow in a cross-section of the channel.

C = an empirical coefficient depending in the main upon the roughness of the solid surfaces of the channel, and also upon the velocity of flow, upon the hydraulic radius, and possibly upon the slope of the water surface.

R = the mean hydraulic radius = the area of the cross-section of the stream of water divided by its wetted perimeter.

S = the slope of the water surface.

This Chezy formula is adopted as a part of the basis for the following derivation of the relation between the wind and the slope of the water surface ultimately produced by it.

Consider the conditions on a lake during a period when a wind of uniform constant velocity and direction is blowing over the surface of the lake. Consider any strip of surface of the lake, of unit width, with the axis of the strip parallel to the direction of the wind. So long as the wind remains constant in velocity and direction it is evident that the rate at which the surface water will be delivered to leeward along the strip by the action of the wind will be approximately constant. Let the volume of water delivered per unit of time past any line across the strip under the action of a wind of fixed velocity be called Q.

The velocity of this surface drift caused by the wind is evidently a maximum at the surface where the wind acts upon the water, diminishes gradually with increase of depth below the surface, and becomes zero, or practically so, at a moderate depth below the surface. This is illustrated by figure 3, plate 4, in which the arrows are proportional to the velocities under con-

sideration. The exact relation between these various arrows is not claimed to be known, and for the purpose of this investigation it is not necessary to know it. The illustration figure 3 is meant merely to show the general conception.

As the wind delivers water toward the lee shore, as an east wind would deliver water along the line BB, figure 1, plate 4, on Lake Erie toward the western shore of the lake, it raises the elevation of the water surface on that leeward portion of the lake. In due time, if the wind remains constant in velocity and direction, a surface slope will be established everywhere on the strip, along BB, downward to windward such that the return current to windward, set up by the action of gravity, will deliver the water to windward at the same rate, Q, that the wind delivers it to leeward. During the continuance of this steady régime, after it is once established, the elevation of the water surface at every part of the strip will remain fixed, the volume of water delivered by the wind past each point of the strip per unit of time to leeward will be Q, and the volume delivered per unit of time past any point by the return gravity current to windward will also be Q.

The return current is illustrated by figure 4, plate 4. As this current is produced by gravity, the Chezy formula (52) expresses the relation necessarily existing between the velocity and the slope. This return current, being produced by gravity, extends throughout the depth, and with sufficient accuracy for the present purpose may be considered as being the same at all depths as shown in the illustration.

As the wind current indicated in figure 3, plate 4, to leeward and the return gravity current to windward, figure 4, exist at the same time, the actual velocities of the water at various depths are properly represented by figure 5, in which each arrow is the algebraic sum of the two corresponding arrows in figures 3 and 4. During the steady régime under consideration the sum of the arrows in figure 3 is the same as the sum of the arrows in figure 4, and in figure 5 the sum of the arrows which point to the left is the same as the sum of the arrows which point to the right. According to these figures, the net delivery of water past the point is zero, the separate volumes delivered per unit of time in opposite directions each being Q.

As the Chezy formula (52) holds for the return current produced by gravity, the following relations (53) to (59) are true during the continuance of the steady régime:

At each point of the strip of unit width under consideration,

$$R = D = \text{depth of water at that point of the strip.} \tag{53}$$

This follows from the fact that the only part of the perimeter of the cross-section of the stream under consideration which encounters resistance at a solid boundary is the bottom. The two sides of the stream are in contact with water of adjacent strips. The strip being of unit width and the wetted perimeter unity, the area of the cross-section of the stream flowing along the strip is numerically the same as the depth.

Under the conditions stated,

$$V = \frac{Q}{\text{area of cross-section of the stream}} = \frac{Q}{D} \tag{54}$$

By substitution from (53) and (54) in (52) there is obtained

$$\frac{Q}{D} = C(DS)^{\frac{1}{2}}$$

From the foregoing equation by solution for S there is obtained

$$S = \frac{Q^2}{C^2}\left(\frac{1}{D^3}\right) \tag{55}$$

in which $\frac{Q^2}{C^2}$ is a constant, during the steady régime under consideration, which will be called C_1.

This constant, $C_1 = \frac{Q^2}{C^2}$, depends on the influences which fix C, the Chezy coefficient, and upon the action of the wind upon the water, which fixes Q.

Equation (55) may for the present purpose be conveniently written thus:

$$S = C_1\left(\frac{1}{D^3}\right) \tag{56}$$

Let the difference of elevation of the surface of the water at any two points along the axis of the strip under consideration be called Hd and let the distance between the two points be called L.

Then

$$Hd = SL \tag{57}$$

From (56) and (57), for any short portion of the strip on which D is constant,

$$Hd = C_1\left(\frac{L}{D^3}\right) \tag{58}$$

In general, under the influence of a steady wind, the surface of the water along any strip during the steady régime will have some such shape as that shown in the curve labeled "disturbed water surface" in figure 1, plate 4. On that curve, which is drawn for a strip on Lake Erie along the line BB and for a west wind, the surface of the western part of the strip is shown depressed below, and along the eastern part elevated above, the normal elevation which it would have if no wind were blowing. One point on the curve, there shown at 1,027 thousands of feet west of the Buffalo gage, is shown as unchanged in elevation by the wind. Call such a point the nodal point of the strip.

Let H be the total disturbance of elevation of any point on the strip under consideration.

Then, from (58), it is evident, if one thinks of a step-by-step integration of differences of elevation from the nodal point of the strip to the point under consideration, that

$$H = C_1 \Sigma \frac{L}{D^3} \tag{59}$$

The summation, indicated by Σ in this formula, of terms each of the form $\frac{L}{D^3}$ is supposed to be made in portions of length L so short that along each portion D (the depth), may be considered constant. The summation is supposed to extend over the whole distance from the nodal point to the point under consideration.

Note that if the point under consideration is at a gage, H is the disturbance of the elevation of the water surface at the gage which is produced by the wind.

DERIVATION OF Σ_x.

In what has preceded, culminating in formula (59), only a single strip, of unit width, parallel to the direction of the wind has been considered.

It is evident, if one reviews the various steps by which (59) was derived, that it is true for strips of any one width within which there is no transverse variation of depth. The slope of the water surface along a strip is independent of the width of the strip provided the depth of water is the same for the whole width of the strip.

Consider an actual lake to be divided up into strips of moderate width, each parallel to the wind direction. Equation (59) applied to each strip independently would show the values of H differing at adjacent points on adjacent strips. Clearly, if such a condition actually existed for a moment the water would begin to flow across the arbitrarily assumed (but non-existent) boundary between strips, from the higher of the two points to the lower. Such cross-currents would tend always to bring adjacent parts of the various strips to one elevation, and thereby tend to modify somewhat the slope along each strip.

In any actual lake, with its irregular depths and irregular shore line, there will certainly be such cross-currents. A study of the complicated relations involved leads to the conclusion that such cross-currents are sluggish in general except in shallow water near shore, and that assumption No. 4, stated below, is near the truth and leads to errors so small as to be negligible in comparison with other errors of this investigation which are unavoidable.

ASSUMPTION NO. 4.

It is assumed that the return gravity current has small components at right angles to the wind direction that cross the boundaries between strips in such a manner as to bring adjacent portions of the various strips nearly to the same elevation and that the slope on each strip thereby established is the same as if the depth everywhere along each strip were the same as the mean depth for all abreast portions of all strips. For example,

if the strip under consideration lay along the line BB in figure 1, plate 4, on the actual Lake Erie, the slope at point F on the strip is that fixed by formula (56) if one uses for D not the depth at F but the mean depth along the line CC', which is at right angles to BB through the point F.

Under assumption No. 4, formula (59) applies to any strip parallel to the wind direction on any actual lake provided one uses for each point on the strip a depth D which is a mean depth along such a line as CC' at right angles to the strip through the point.

The values of Σ_x, the summation indicated in (59), have been computed for 8 wind directions for the Buffalo and Cleveland gages on Lake Erie and for the Milwaukee, Harbor Beach, and Mackinaw gages on Lake Michigan–Huron. The way in which formula (59) and assumption No. 4 have been applied in these computations will be illustrated by selected portions of the computations.

EXAMPLE OF COMPUTATION OF Σ_x.

The whole area of Lake Erie, as shown on a chart of Lake Erie issued by the United States Lake Survey, was divided up into strips parallel to CC' on figure 1, plate 4, the axis of each strip being in the meridian. The widths of the strips varied from 2,000 feet to 50,000 feet. The strips were wide where the depths were large and comparatively regular, and were narrow in shallow water and where the depths were irregular. The division lines having been drawn on the chart, the separate depths were estimated from the chart and entered in the computation illustrated by selected portions in table No. 8.

In table No. 8 the strip limits are given, in the first column, in thousands of feet measured westward from the Buffalo gage. The first strip shown in the table has for its eastern limit a meridian line which is 50,000 feet west of the Buffalo gage, and for its western limit one which is 100,000 feet west of the Buffalo gage. The approximate location of each of the strips may be seen on figure 1 of plate 4, on which a scale of distances westward from the Buffalo gage is shown.

With the chart before one, and with dividers in hand, it is a matter of easy routine to visually divide any strip across the lake into ten equal parts, to estimate the mean depth in each part, and to enter it in the computation as shown in the second to the eleventh columns of table No. 8. The mean depth shown on each line in the twelfth column is the mean of the depths entered in the next preceding ten columns on that line.

The values of D^3 as shown in table No. 8 are rounded off to three significant figures. That gives sufficient accuracy.

Each L is a distance parallel to the wind under consideration—an east wind in this case. It is in each case the same as the width of the strip, which is the difference between the two figures given in the first column as strip limits. For convenience, L is given in thousands of feet in the table.

The first group of strips shown in table No. 8 includes the deepest part of

Lake Erie, southeast of Long Point. The depths are comparatively regular and the strips are 50,000 feet (nearly 10 miles) wide. Note the relatively small values of $\frac{L}{D^3}$, even though values of L are large.

The second group of strips shown in table No. 8 includes that part of the

TABLE No. 8.—*Computation of* $\frac{L}{D^3}$ *for E–W axis, Lake Erie.*

Strip limits.		Mean depth in fathoms in each tenth of strip from north to south.										Mean depth.		D^3	L	$\frac{L}{D^3}$
		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	10th.	*fath.*	*feet.*			
50 to	100	4	10	11	11	12	11	12	11	12	9	10.3	62	238,000	50	.210
100	150	5	9	10	13	14	15	16	17	15	9	12.3	74	405,000	50	.123
150	200	6	10	14	16	22	24	23	22	16	14	16.7	100	1,000,000	50	.0500
200	250	4	12	17	23	31	30	26	22	18	10	19.3	116	1,560,000	50	.0321
250	300	5	12	19	26	32	29	24	20	15	8	19.0	114	1,480,000	50	.0338
650	700	7	11	12	13	13	14	14	13	12	7	11.6	70	343,000	50	.146
700	750	8	11	13	14	14	14	14	13	11	8	12.0	72	373,000	50	.134
750	755	7	11	13	14	14	14	13	12	11	8	11.7	70	343,000	5	.0146
755	758	7	11	12	14	14	14	13	12	11	8	11.6	70	343,000	3	.00874
758	800	8	11	12	14	14	14	13	12	11	9	11.8	71	358,000	42	.117
1,200	1,205	0	1	2	3	3	3	2	2	1	1	1.8	11	1,330	5	3.76
1,205	1,210	0	2	2	3	3	2	1	1	1	1	1.6	10	1,000	5	5.00
1,210	1,215	0	2	2	3	3	1	1	1	1	1	1.5	9	729	5	6.86
1,215	1,220	0	2	2	2	2	2	1	1	1	0	1.3	8	512	5	9.77
1,220	1,225	1	2	2	2	1	1	1	1	1	0	1.2	7	343	5	14.6

lake which is near Cleveland. In this region the bottom of the lake is nearly level over about nine-tenths of the width of the lake, the depth being from 10 to 14 fathoms except near each shore. The shape of the cross-section of the lake in this region is illustrated by figure 2, plate 4, which is drawn to scale. Two of the strips are 50,000 feet wide. The other three strips were made narrower simply for the purpose of making one strip boundary run through the Cleveland gage, which is 758,000 feet west of the Buffalo gage.

The third group of strips shown in table No. 8 lies in the very shallow portions of the lake near its western end. The meridian, which is 1,200,000 feet west of Buffalo gage, passes between the Toledo harbor light, near the outer end of the dredged channel to Toledo, and Cedar Point on the south shore of the lake. The extreme western end of the lake is 1,239,000 feet west of Buffalo gage. Note that in this group of strips the depths vary from 3 fathoms to less than one-half of a fathom (which is shown as zero in the table). Note that L is small in this group, only 5,000 feet, and that nevertheless the values of $\frac{L}{D^3}$ are relatively large, -14.6 for the last strip shown.

Sandusky Bay was covered by a computation, similar to that illustrated by table No. 8, separately from the main computation which covered the lake. This was the only case so treated for east or west winds across Lake Erie. In general, it was found necessary to make such separate computations for bays or portions of bays which are not covered by continuous strips across the lake transverse to the assumed wind, the bay being in each such case cut off from the lake on each strip by an intervening point of land. In some of the computations made for certain directions of wind, and especially in Lake Michigan–Huron, there were several such special auxiliary computations for bays.

When such a computation as that illustrated by table No. 8 had been completed for a given assumed wind and lake, a computation such as that illustrated in table No. 9 was made.

The strip limits are given in the first column of table No. 9 in thousands of feet measured westward from the Buffalo gage, just as they were given in table No. 8.

Each value of L corresponding to the width of a strip is given in thousands of feet.

The third column of table No. 9 gives the length of each strip measured at right angles to the assumed wind in thousands of feet. The entry 110 in the third column for the strip 50 to 100 in table No. 9 means that said strip is 110,000 feet long from the Ontario (Canadian) shore to the Ohio shore.

The product of the width of any strip as shown in the second column by the length of that strip as shown in the third column is its area as shown in the fourth column, expressed in units of 1,000,000 square feet.

Table No. 9.—*Computation of* $\Sigma\frac{L}{D^3}$ *for E–W axis, Lake Erie.*

Strip limits.		L	Length of strip.	Area of strip.	$\Sigma\frac{L}{D^3}$ to far side of strip.	$\Sigma\frac{L}{D^3}$ to middle of strip $=(b)$.	(b) times area.	Continuous sum of preceding column.
......	......	....	....	....	....	....	....	....
......	......	....	....	....	....	....	....	....
50 to	100	50	110	5,500	160	160	880,000	1,450,000
100	150	50	144	7,200	160	160	1,150,000	2,600,000
150	200	50	177	8,850	161	161	1,425,000	4,030,000
200	250	50	207	10,350	161	161	1,670,000	5,700,000
250	300	50	220	11,000	161	161	1,770,000	7,470,000
......	......	....	....	....	....	....	....	....
......	......	....	....	....	....	....	....	....
650	700	50	332	16,600	162	162	2,690,000	24,900,000
700	750	50	346	17,300	162	162	2,800,000	27,700,000
750	755	5	342	1,710	162	162	277,000	28,000,000
755	758	3	340	1,020	162	162	165,000	28,200,000
758	800	42	319	13,400	162	162	2,170,000	30,300,000
......	......	....	....	....	....	....	....	....
......	......	....	....	....	....	....	....	....
1,200	1,205	5	66	330	185	183	60,400	44,400,000
1,205	1,210	5	68	340	190	188	63,900	44,400,000
1,210	1,215	5	66	330	196	193	63,700	44,500,000
1,215	1,220	5	56	280	206	201	56,300	44,500,000
1,220	1,225	5	48	240	221	113	27,100	44,600,000
......	......	....	....	....	....	....	....	....
......	......	....	....	....	....	....	....	....

In the fifth column there is shown the continuous summation of $\frac{L}{D^3}$ step by step from the extreme eastern part of Lake Erie, near Buffalo, to the far side of the strip. The values of $\Sigma\frac{L}{D^3}$ shown in this fifth column were obtained by adding successively the values of $\frac{L}{D^3}$ computed as shown in table No. 8. For example, $\Sigma\frac{L}{D^3}$ for the west (or far) side of strip 1,200 to 1,205 is shown as 185 in the fifth column of table No. 9. The value of $\frac{L}{D^3}$ for strip 1,205 to 1,210 is shown as 5.00 in the last column of table No. 8. In the fifth column of table No. 9 the value of $\Sigma\frac{L}{D^3}$ for the west (or far) side of strip 1,205 to 1,210 is shown as 190 (which is $185+5.00$).

Each value in the sixth column of table No. 9, called (b) for convenience, is the mean of the values of $\Sigma\frac{L}{D^3}$ at the two sides of the strip as shown in the fifth column.

Each value shown in the seventh column is the product of the two quantities in that line entered in the fourth and sixth columns. Each value in the seventh column is rounded off to three significant figures, as that gives sufficient accuracy.

The continuous sum shown in the last column of table No. 9 was started at the extreme eastern part of Lake Erie and carried forward continuously, and was rounded off to three significant figures after the summation had been made continuously.

DETERMINATION OF POSITION OF NODAL LINE.

The values of $\Sigma\frac{L}{D^3}$ shown in the fifth and sixth columns of table No. 9 are all referred to the extreme eastern part of Lake Erie, near Buffalo. For use in formula (59) it is necessary to have these summations referred to the nodal point on the line of disturbed elevations of the water surface, such as is indicated by the point G in figure 1, plate 4. Hence, the location of this point G must be determined.

Figure 1, plate 4, is drawn for a west wind. It shows the disturbed water surface depressed to the westward of G and elevated to the eastward. It is clear that if the wind were from the east the disturbed water surface would be depressed to the eastward of the nodal point, G, and elevated to the westward. The ultimate effect of a steady east wind is to transfer water from portions of the lake lying to the eastward of the nodal point to portions of the lake lying to the westward of that point.

In the computation of which selected parts are shown in table No. 9 the summations shown in the fifth, sixth, and eighth columns are all referred to an initial point at the extreme eastern part of the lake at the water surface. This point is the lowest point of the disturbed water surface under the influence of an east wind. Let the total disturbance of elevation of the water by an east wind at this particular point, which has been used as the initial point, be called Hi.

The total amount of water in the lake which is above the elevation of this initial point in the disturbed condition under the influence of an east wind is

$$\Sigma\ (\text{area of strip})\left(C_1\Sigma\frac{L}{D^3}\right) \tag{60}$$

In (60) the outer Σ stands for a summation, strip by strip for the strips shown in the computations illustrated in tables Nos. 8 and 9, of the products of the area of each strip by the quantity $C_1\Sigma\frac{L}{D^3}$ for that strip. This summation is supposed to start at the initial point and to extend over the whole lake. So, too, the summation $\Sigma\frac{L}{D^3}$ for each strip is supposed to start at the same initial point, but is supposed to stop for each strip at that

strip. From (58) it is clear that the quantity $C_1\Sigma\frac{L}{D^3}$ at each strip is the mean elevation of the water surface of that strip (in the disturbed condition) referred to the initial point as a zero. Hence, the product of the area of the strip into $C_1\Sigma\frac{L}{D^3}$ for that strip is the amount of water in that strip which lies above the elevation of the initial point, and it is clear that the grand summation indicated in (60) is the total amount of water in the lake which is above the initial point during the disturbed condition.

When the surface of the lake is undisturbed its whole surface is at one elevation, and the total amount of water in the lake then lying above the elevation of the initial point under consideration is

$$(Hi)\ (\text{area of lake}) \tag{61}$$

A wind blowing the surface of the lake does not change the total content of the lake. Hence, the total amount of water above the initial point (which was the lowest point of the disturbed surface) must be the same in the disturbed and the undisturbed condition. Hence, (60) and (61) are equal. By placing them so in an equation and solving for Hi there is obtained

$$Hi=\frac{\Sigma\ (\text{area of strip})\left(C_1\Sigma\frac{L}{D^3}\right)}{\text{area of lake}}=\frac{C_1\Sigma\ (\text{area of strip})\left(\Sigma\frac{L}{D^3}\right)}{\text{area of lake}} \tag{62}$$

Equation (62) is an expression for the disturbance of elevation of the water surface at the initial point referred to the nodal point, of which the location is as yet unknown. C_1 may be taken outside both summation signs as shown because it is a constant which is the same for all strips.

Equation (59) is an expression for the disturbance of elevation of any point on the lake. Hence, applying this formula (59) to the initial point under consideration, there is obtained

$$Hi=C_1\Sigma\frac{L}{D^3} \tag{63}$$

in which the summation $\Sigma\frac{L}{D^3}$ extends from the nodal point to the initial point.

By placing the two expressions for Hi of (62) and (63) equal to each other and dividing each side of the equation by C_1 there is obtained, as an equation applicable at the nodal point only,

$$\Sigma\frac{L}{D^3}=\frac{\Sigma\ (\text{area of strip})\left(\Sigma\frac{L}{D^3}\right)}{\text{area of lake}} \tag{64}$$

Equation (64) identifies the value of $\Sigma\frac{L}{D^3}$ as shown in the fifth and sixth columns of the computation illustrated in table No. 9, at which the nodal point lies. In table No. 9 the seventh column shows the value for each strip of one of the products indicated in the numerators of the second member of (64). The continuous summation of these products is shown in the eighth column. This summation (eighth column) at the end of the computation, covering the whole lake, is the numerator of the second member of (64). The denominator of the second member of (64), the area of the lake, is the sum of the separate areas of the strips as shown in the fourth column of table No. 9.

In the concrete case illustrated in table No. 9 the value of the numerator of the second member of (64) was found to be 45,500,000. The area of the lake was found from the computation to be 276,000. Hence, the value of $\Sigma\frac{L}{D^3}$ at the nodal point was found to be $\frac{45{,}500{,}000}{276{,}000}=165$. By inspecting the fifth and sixth columns of the computation illustrated in table No. 9 it was found that this value of $\Sigma\frac{L}{D^3}$ occurred at 1,027,000 feet west of the Buffalo gage and that therefore the nodal point and nodal line are in that location, as indicated in figure 1, plate 4.

The preceding derivation of certain formulæ and of the location of the nodal point has been written for a definite case, for an east wind over Lake Erie, and for an initial point at the extreme eastern part of Lake Erie, where the water will be lowest under the influence of an east wind. The demonstration and the corresponding methods of computation are of general application, with certain obvious minor changes in statement, for any lake, for any initial point on that lake, and for any wind direction.

The computations have been made for eight wind directions for Lake Erie and for Lake Michigan–Huron.

As already noted (page 46), in making the computations for an east wind over Lake Erie it was found that a special auxiliary computation of the general character shown in table No. 8 must be made for Sandusky Bay. In the corresponding auxiliary computation for Sandusky Bay of the character illustrated by table No. 9 the computation was started with the value of $\Sigma\frac{L}{D^3}$ found at the entrance of the bay, at the first strip in the bay which is found to be cut off from the corresponding strip in the main lake by intervening land.

Many such auxiliary computations were found to be necessary for bays cut off from the main lake by intervening land, so far as certain strips of the character used in the computation are concerned.

In what precedes, the expression "nodal point" has frequently been used, having in mind a profile view of the water surface at right angles to the

assumed wind direction. Such a view is indicated in figure 1, plate 4. A nodal line passes through the nodal point of the profile and extends across the lake. Under the action of a wind, the nodal line on the surface of the water remains unchanged in elevation, all parts of the lake surface to leeward of the nodal line are raised, and all parts to the windward are lowered.

POSITIONS OF VARIOUS NODAL LINES.

The positions of the various nodal lines for various wind directions and for Lakes Erie and Michigan–Huron are shown on plates 2, 5, and 6. Each nodal line appertains to a wind direction at right angles to that line.

It will be noted that in various cases there is a short portion of a nodal line extending across a bay, which portion is parallel to but not in line with the portion of the nodal line which is in the main lake. In each of these cases the value of $\Sigma\frac{L}{D^3}$ which identifies the nodal line was found by examinations of the computation to exist in the bay. These are true nodal lines fixed at certain locations in certain bays, due to the fact that the slopes of the water surface in those bays are as shown in formula (56), and the elevation of the water surface in the mouth of the bay, where it is first cut off from the lake by intervening land, is as indicated by formula (59).

On Lake Erie (see plate 2) the nodal line for east and west winds, a line which therefore runs north and south, is much nearer to the western end of the lake than to its middle. It is in the longitude of Pelee Island. Note that when the wind becomes northwest or southeast the main portion of the nodal line still remains in this locality, running, of course, in the southwest–northeast direction, but that it has two detached bay portions, one in the bight northeast of Point Pelee and the other in the bight northeast of Point aux Pins. For north winds and south winds the main portion of the nodal line is in a central position across Lake Erie, but a bay portion extends westward from near the end of Point Pelee. For northeast winds and southwest winds the nodal line is again far from the central portion of the lake—in the southwest part. The crowding of three of the four nodal lines toward the western end of Lake Erie is due to the relative shallowness of that portion of the lake which is west of Pelee Island. There the depth is less than 6 fathoms, as a rule, whereas in much of the main portion of the lake the depth is from 10 to 14 fathoms.

On Lake Michigan–Huron (see plates 5 and 6) the main portion of the nodal line for southwest winds and northeast winds lies in Lake Michigan, not far to the westward of the Strait of Mackinac, and has two detached bay portions, one across Saginaw Bay and one near Port Huron. The main portion of the nodal line for west winds or east winds is in the same locality in Lake Michigan, not far to the westward of the Strait of Mackinac, and has a single detached bay portion across Saginaw Bay. The main portion of the nodal line for northwest winds and southeast winds is in Lake Huron,

far to the southeastward of the Strait of Mackinac. It has no detached bay portions worthy of note. For north winds and south winds the nodal line consists of three short detached portions, one near the extreme southern end of Lake Michigan, one across Saginaw Bay, and one in the extreme southern end of Lake Huron, near Port Huron. Note the range horizontally through which the nodal line shifts when the wind changes from northwest to north and again when it changes from north to northeast.

There are some other very short detached bay portions of the nodal lines on Lake Michigan–Huron, but they are of local importance only. They can not be shown on the scale of plates 5 and 6.

To apply formula (59) to a particular gage, the Buffalo gage for example, let it be rewritten thus:

$$H_b = C_1 \Sigma_b \tag{65}$$

in which the subscripts b stand for the Buffalo gage, and Σ_b is the $\Sigma \frac{L}{D^3}$ of formula (59) from the nodal line to the Buffalo gage. The value of Σ_b may be obtained easily from such a computation as that illustrated in table No. 9 by merely taking the difference between $\Sigma \frac{L}{D^3}$ at the nodal line, of which the value is determined from this computation as already indicated, and the value of $\Sigma \frac{L}{D^3}$ at the Buffalo gage which is shown directly in the computation.

Formula (65) is of the proper form for application to any gage on any lake by merely changing the subscript and making the corresponding changes in interpretation.

The values of the Σ of such an equation as (65) will in general be different for each gage and for each direction of wind over the lake.

VALUES OF Σ_x.

The various values of Σ_x have been computed, in the manner indicated on pages 44–52, for the five gages at Buffalo, Cleveland, Milwaukee, Harbor Beach, and Mackinaw. The values are shown in table No. 10, which follows, in the units indicated in connection with tables Nos. 8 and 9.

There are certain features in table No. 10 which it is desirable to note in the attempt to secure definite and correct ideas as to the wind effects on the two lakes under consideration. Attention is called to some of these in the following paragraphs. Consult plates 2, 5, and 6 in connection with these paragraphs.

The values of Σ_x are in general much larger for the two Lake Erie gages than for the three Lake Michigan–Huron gages. The smallest value of Σ_x, at either Buffalo or Cleveland, shown in the table is 1.72 for north and south winds at Cleveland. The largest value of Σ_x at any of the three Lake Michigan–Huron gages is 0.95 for west and east winds at Milwaukee. Hence,

one must expect to find the wind effects much smaller at these Lake Michigan–Huron gages than at these Lake Erie gages.

TABLE No. 10.—*Values of Σ_x for use in formula (59) at the gages indicated.*

Direction of wind.	At Buffalo.	At Cleveland.	At Milwaukee.	At Harbor Beach.	At Mackinaw.
NE...........	−8.32	−3.89	+0.16	−0.86	−0.64
E.............	−5.90	−2.86	+ .95	− .77	− .41
SE............	−2.26	−1.90	+ .25	− .05	+ .15
S.............	+3.45	−1.72	+ .36	+ .35	+ .44
SW...........	+8.32	+3.89	− .16	+ .86	+ .64
W............	+5.90	+2.86	− .95	+ .77	+ .41
NW...........	+2.26	+1.90	− .25	+ .05	− .15
N.............	−3.45	+1.72	− .36	− .35	− .44

The average values of Σ_x at the five gages stand in the order in which the gages are given in table No. 10. Buffalo has the largest average value of Σ_x, and hence the largest wind effects, among these stations, and Mackinaw has the smallest.

Buffalo is at the eastern end of Lake Erie, and Cleveland is to the westward of the middle of the lake. Cleveland is on the south shore of the lake, and Buffalo is near the most northerly point of the lake. From these facts alone one would infer that in general the wind effects would be of opposite signs at the two gages. Yet table No. 10 shows the wind effects to be the same in sign at the two gages for six out of the eight directions tabulated. The only values of Σ_x at the two gages which are of opposite signs are those for the directions south and north. This apparently anomalous agreement in signs arises from the crowding of the nodal lines into the western portion of Lake Erie, in the region of shallow water, to which attention was called on page 51, in such wise that for all directions of wind except north and south Cleveland and Buffalo are on the same side of the nodal line. The effect of southwest, west, and northwest winds is to raise the elevation of the water surface at Cleveland rather than to lower it. One would naturally expect the latter from consideration of the plan of the lake alone.

There is a large change in Σ_x at Cleveland when the wind changes from south to southwest, viz., from −1.72 to +3.89. Note by comparison with plate 2 that this is due to the sudden shift of the nodal line from the leeward of Cleveland to the windward when this change of wind occurs. The corresponding statements are true for the shift of wind from north to northeast.

Milwaukee is much nearer the southern end of Lake Michigan than the northern end. From this fact alone one would naturally infer that a south wind would lower the elevation of the water surface at Milwaukee. But table No. 10 shows that Σ_x for Milwaukee for a south wind is +0.36 and that therefore the water surface is raised at Milwaukee by a south wind.

Note that this is due to the fact that the nodal line for a south wind is near the extreme southern end of Lake Michigan, as shown on plate 5.

The same apparently anomalous condition is also found at Harbor Beach (see plate 6). For a south wind the nodal line is near to Port Huron, at the extreme southern end of Lake Huron. Σ_x for a south wind is $+0.35$ at Harbor Beach, and a south wind raises the water surface at Harbor Beach.

THE WIND EXPONENT.

The greater the velocity of the wind blowing over a lake the more rapid will be the drift of the surface water to leeward. The larger the surface drift to leeward the greater will be the return current to windward ultimately produced by gravity after the steady régime has been established. The greater the return current the steeper will be the surface slopes of the water and the greater the disturbance of elevation of the surface at any given point for a wind from a given direction.

As a first approximation, it might be assumed that the rate at which the wind delivers water to leeward in the surface drift is proportional to the velocity of the wind. If so, the velocity of the return gravity current to windward would be proportional to the wind velocity. As shown by the Chezy formula, (52), page 40, the surface slope for a steady current produced by gravity is proportional to the square of the velocity of the current. The wind effects, as disturbances of elevation of the water surface at a gage, are proportional to the slopes. Hence, on the first approximation suggested, the wind effects would be proportional to the squares of the wind velocities, and the exponent of h in the wind-effect formula (51), page 39, would be 2.0.

But the validity of the approximate assumption suggested is decidedly uncertain. So far as the writer knows, there is no proof, theoretical or observed, that the drift of the water to leeward at the very surface is proportional to the wind velocity, though that seems to be a plausible assumption.

The depth to which the surface drift extends is probably a function of the wind velocity. With higher wind velocities, the drift at the very surface will certainly be more rapid, and the depth to which the drift extends will probably be greater than during light winds. If this is the case, then the total drift to leeward in a given wind, expressed in volume per unit of time, the Q of page 40, will be a different function of the wind velocity than is the velocity of the drift at the very surface.

The character of the water surface on which the wind acts to drive the water to leeward varies greatly for different wind velocities. It varies greatly in roughness, and the roughnesses are themselves in motion at rates which are in various relations to the wind velocities. During very light winds the water surface is relatively very smooth, broken only in general by wind ripples. With moderate winds, say 10 to 15 miles per hour, there is a decided roughness in the form of wind waves, which are traveling to leeward at a rate not differing greatly from the velocity of the wind. The wind has a

much lighter grip upon or impact upon these roughnesses than it would have if the roughnesses were stationary. The impact of the wind on the rear surface of a moving wave is certainly rather light when the wave is moving nearly as fast as the wind. During very high winds, say in excess of 40 miles per hour, the wind waves are high and expose a very rough surface to the action of the wind, this roughness is traveling to leeward at a much slower rate than the wind moves, and the drift of water to leeward is now augmented by the large throw to leeward of the upper part of each wave as it breaks.

From such considerations as are indicated in the preceding few paragraphs it appears that the wind exponent in such a formula as (51) is probably 2 (or more), but that the theory is so uncertain that it is best to derive the exponent from observations rather than from theory.

Accordingly, in early stages of the investigation of wind effects, a number of least-square solutions were made in pairs, in which the two solutions of a pair differed only or mainly in the exponent assigned to h, the wind velocity. These solutions indicated that the more nearly the exponent was made to approach to 2.4 from either side the closer was the agreement obtained between the computed results and the observed facts. In other words, more accurately stating the matter, the sum of the squares of the residuals appeared to be a minimum when the exponent 2.4 was used. A residual in this statement is a discrepancy between the computed change of elevation of the water surface at a gage during a given interval, on the one hand, and the observed change of elevation recorded by the gage during that interval, on the other hand.

Hence, the exponent 2.4, derived thus from observations, was adopted for the final formula for wind-effect investigations, as shown in (51), page 39.

Some further information is given at an appropriate place later in this publication, in the discussion of errors of computed wind effect, as to the particular least-square solutions which served to establish 2.4 as the most probable value of the exponent and as to the estimated accuracy of that exponent.

At this point attention is called especially to the fact that the value 2.4 is derived from observation rather than theory. It is the investigator's belief, based on much study of the details of the investigation and some theoretical considerations, that an error of moderate amount in the adopted exponent has but slight effect on the final outcome of the investigation expressed in terms of daily corrections for wind effect, provided the exponent, once adopted, is used consistently throughout the remainder of the investigation.

EXAMPLE OF OBSERVATION EQUATIONS FOR WIND EFFECTS.

The least-square solutions for determining the wind effects are based upon hourly observations of the water surface and upon formula (51), shown on page 39.

The form of each observation equation is as follows:

$$\left[\left(\frac{h^{2\cdot4}}{100}\right)_p(\Sigma_x)-\left(\frac{h^{2\cdot4}}{100}\right)_c(\Sigma_x)\right]C_p+\left[\left(\frac{h^{2\cdot4}}{100}\right)_c(\Sigma_x)-\left(\frac{h^{2\cdot4}}{100}\right)_{c+1}(\Sigma_x)\right]C_a+L=v \quad (66)$$

h is the wind velocity, in miles per hour, at the station to which the equation refers, ending at the hour specified by the subscript.

The subscript c refers to the current hour—the hour by which the equation is designated. The subscript p refers to the preceding hour. The subscript $c+1$ refers to the hour following the current hour.

C_p and C_a are unknown constants to be derived from the observations by means of the least-square solution.

L is the elevation of the water surface at the gage at the current hour minus the elevation of the water surface at the gage at the preceding hour, the elevations being first corrected for any known effects for which it is feasible to apply reliable corrections. The only such correction applied in this investigation was the correction for hourly barometric effects, computed as indicated on pages 32–36.

The symbol Σ_x stands for the appropriate value from table No. 10 for the gage station under consideration—Buffalo gage, Cleveland gage, etc.—and for the wind direction at that station for the hour specified by the subscript of the term $\left(\frac{h^{2\cdot4}}{100}\right)$ into which this particular Σ_x is multiplied.

Compare (66) with (51) on page 39. It appears that if C_p, an unknown constant to be determined, is considered to be the C_x of equation (51), then $\left(\frac{h^{2\cdot4}}{100}\right)_p(\Sigma_x)C_p$ in (66) is the effect of the wind at the preceding hour in elevating the surface of the water at the gage. Similarly, $\left(\frac{h^{2\cdot4}}{100}\right)_c(\Sigma_x)C_p$, on the same supposition, is the effect of the wind at the current hour in elevating the surface of the water at the gage. The difference shown as the first of three terms in the first member of the observation equation (66), namely,

$$\left[\left(\frac{h^{2\cdot4}}{100}\right)_p(\Sigma_x)-\left(\frac{h^{2\cdot4}}{100}\right)_c(\Sigma_x)\right]C_p$$

is the computed fall in the water surface from the preceding to the current hour on the supposition that C_p is the C_x of equation (51).

One modification should be made of the statement in the preceding paragraph, in which it is implicitly assumed that there is no lag in the response of the water to a change of wind. It should be noted that h, as defined just below equation (66), is the wind velocity at the hour ending at the time specified. If, for example, the hour specified is 10 a.m., the velocity h is the velocity for the hour from 9 a.m. to 10 a.m. This velocity is in fact ordinarily obtained by counting up the number of miles of wind, as shown on an automatic record from an anemometer, that passed the record-

ing station between 9 a.m. and 10 a.m. In such a case the velocity applies more strictly to 9.30 a.m. than to 10 a.m., or, in other words, it is the velocity of one-half hour before the time specified. So the statement of the preceding paragraph would apply strictly if the wind effect lagged one-half hour behind each change in the wind.

Compare the second of the three terms in the first member of observation equation (66), namely,

$$\left[\left(\frac{h^{2\cdot 4}}{100}\right)_c(\Sigma_x)-\left(\frac{h^{2\cdot 4}}{100}\right)_{c+1}(\Sigma_x)\right]C_a$$

with equation (51). It appears from the comparison that in the same manner that the first term expresses the computed fall in the water surface if the lag is one-half an hour, so the second term would properly express it if instead of lag there is an anticipation of one-half an hour. An anticipation means in this case merely that the water surface at the gage station changes before the change in wind occurs at the Weather Bureau station at which the wind is recorded, which may be a mile or even several miles away, and not that the effect on the water preceded the cause which produced it. It is desirable, also, if one tends to be skeptical of an anticipation in the sense indicated in the formula, to consider that an effect at the gage may precede the arrival of the wind change at the gage, since the wave of water piled up by an approaching wind may outrun the progressive change in the wind.

If, then, the least-square solution shows derived values of C_p and C_a which are equal, the meaning is that the wind effect at the gage (a change of elevation of water surface) is, upon an average, simultaneous with the change in the wind at the Weather Bureau station, if C_p is finite and C_a zero the lag is 0.5 hour, and if C_p is zero and C_a finite there is an anticipation of 0.5 hour. For intermediate cases the lag or anticipation has intermediate values.

Studies and various least-square solutions made in this investigation before the final form of the observation equations as shown in (66) was adopted showed that the discoverable lag, if any, in the wind effects is probably very small, a few minutes only. Hence, (66), based on the supposition that the lag is very small, is deemed to be the best form for the observation equations.

If the computed fall of the water surface, represented by the first two terms of the first member of equation (66), were exactly equal to and opposite in sign to the observed rise, L, then the whole first member of (66) would be zero, and the computed residual, v, in the second member would be zero. This would be the case if both the theory and all the observations were absolutely perfect. In the actual case, each v, a discrepancy between computation and observation, is a residual for a particular hour between theory and computation on the one hand and observation on the other. A large group of such residuals from many observation equations furnishes a measure of the accuracy of the theory and the computation based upon it.

The following set of observation equations for August 5, 1910, at Buffalo, serves as a typical example. They are a part of solution W25, which included in all 470 such equations, covering 500 hours out of 22 selected days.

Wind Observation Equations, Buffalo, August 5, 1910, Solution W25.

1 a.m.	$-33C_p-6C_a-29=v_1$	1 p.m.	$-150C_p-100C_a+20=v_{11}$
2	$-6C_p-24C_a+14=v_2$	2	$-100C_p\quad 0C_a+8=v_{12}$
3–4	$-47C_p-11C_a+31=v_3$	3	$0C_p-33C_a-5=v_{13}$
5–6	$-12C_p-24C_a+32=v_4$	4	$-33C_p-83C_a+2=v_{14}$
7	$0C_p\quad 0C_a-3=v_5$	5	$-83C_p+83C_a+17=v_{15}$
8	$0C_p+24C_a-16=v_6$	6	$+83C_p-42C_a+19=v_{16}$
9	$+24C_p-48C_a-14=v_7$	7	$-42C_p+75C_a-21=v_{17}$
10	$-48C_p-70C_a+23=v_8$	8	$+75C_p+374C_a+2=v_{18}$
11 a.m.	$-70C_p-160C_a-21=v_9$	9	$+374C_p+154C_a-30=v_{19}$
N	$-160C_p-150C_a+22=v_{10}$	10	$+154C_p+59C_a-27=v_{20}$
		11 p.m.	$+59C_p-59C_a+26=v_{21}$
		M	$-59C_p-12C_a+11=v_{22}$

The unit used in expressing L, the absolute term, is 0.01 foot.

The basis on which such combinations as are indicated for 3 and 4 a.m. and 5 and 6 a.m. were decided upon will be indicated later in connection with the discussion of the accuracy of the computed wind effects. The basis for certain rejections which were made will also be indicated later in the same place.

Table No. 11, which follows on page 59, shows how the coefficients of C_p and C_a were computed.

The wind velocities and wind directions as shown in the second and third columns of table No. 11 were observed at Buffalo by the Weather Bureau.

The values in the second column, wind velocities in miles per hour, are the values of h from which the fourth column of table No. 11 was computed.

The values of Σ_b shown in the fifth column were taken from table No. 10 for the observed wind directions at Buffalo as shown in the third column.

Each value in the sixth column is the product of the values shown on the same line in the fourth and fifth columns.

Each value of the coefficient of C_p as shown in the seventh column is the difference, in the sense (preceding − current), of the values shown on two lines in the sixth column. Similarly, the coefficient of C_a as shown in the last column of the table is the difference of two values in the sixth column.

A comparison of the seventh and eighth columns of table No. 11 with the coefficients of C_p and C_a in the example of observation equations will make the relation clear. Note that in the combined equation for the two hours 3 and 4 a.m. the coefficient of C_p, -47, is the algebraic sum of the two values for the coefficient of C_p shown in table No. 11 for the hours 3 and 4 a.m., namely, -24 and -23, respectively. In each such case the coefficient in a combined equation is the algebraic sum of the corresponding coefficients of the separate equations which were combined.

TABLE No. 11.—*Buffalo, August 5, 1910.*

	Wind velocity.	Wind direction.	$\frac{h^{2\cdot4}}{100}$	Σ_b	$\left(\frac{h^{2\cdot4}}{100}\right)(\Sigma_b)$	Required coefficients.		
						C_p	C_a	$W.$
								feet.
M.........	14	NW	6	+2.26	+ 14	....	....	....
1 a.m......	16	W	8	+5.90	+ 47	− 33	− 6	+.04
2	17	W	9	+5.90	+ 53	− 6	− 24	+.05
3	20	W	13	+5.90	+ 77	− 24	− 23	+.07
4	22	W	17	+5.90	+100	− 23	+ 12	+.09
5	21	W	15	+5.90	+ 88	+ 12	− 24	+.08
6	23	W	19	+5.90	+112	− 24	0	+.10
7	23	W	19	+5.90	+112	0	0	+.10
8	23	W	19	+5.90	+112	0	+ 24	+.10
9	21	W	15	+5.90	+ 88	+ 24	− 48	+.08
10	25	W	23	+5.90	+136	− 48	− 70	+.12
11 a.m......	30	W	35	+5.90	+206	− 70	−160	+.18
N.........	33	SW	44	+8.32	+366	−160	−150	+.32
1 p.m......	38	SW	62	+8.32	+516	−150	−100	+.45
2	41	SW	74	+8.32	+616	−100	0	+.54
3	41	SW	74	+8.32	+616	0	− 33	+.54
4	42	SW	78	+8.32	+649	− 33	− 83	+.57
5	44	SW	88	+8.32	+732	− 83	+ 83	+.64
6	42	SW	78	+8.32	+649	+ 83	− 42	+.57
7	43	SW	83	+8.32	+691	− 42	+ 75	+.61
8	41	SW	74	+8.32	+616	+ 75	+374	+.54
9	32	W	41	+5.90	+242	+374	+154	+.21
10	21	W	15	+5.90	+ 88	+154	+ 59	+.08
11 p.m......	20	NW	13	+2.26	+ 29	+ 59	− 59	+.03
M.........	21	W	15	+5.90	+ 88	− 59	− 12	+.08

Note that in table No. 11 $\frac{h^{2\cdot4}}{100}$ was 88 at 5 p.m., when the wind velocity was 44 miles per hour, and only 6 at the preceding midnight, when the wind velocity was 14 miles per hour—that is, $\frac{h^{2\cdot4}}{100}$ was increased nearly fifteenfold between midnight and 5 p.m. The increase in the theoretical wind effect would therefore have been nearly fifteenfold if there had been no change in wind direction. With the actual change in wind direction which occurred, from NW to SW, the theoretical wind effect increased more than fifty-two times, as the change in the sixth column of table No. 11 is from 14 to 732. Attention is invited to this case, and in general to an inspection of table No. 11, to secure an appreciation of the complicated law controlling the wind effects. Note the relatively large increases in wind effects produced by moderate increases in high winds, as, for example, the increase from 42 to

44 miles per hour between 4 and 5 p.m. Note, also, the large changes produced by a change of 45° in the wind direction.

The last column of table No. 11, headed *W*, is explained later. It is not intended for consideration at this point.

The observed elevations of the water surface at the Buffalo gage at each hour, as read from the automatic gage record of the Lake Survey, the hourly barometric effects as computed according to pages 32–36, and the corrected elevation of the water surface at each hour as it would have been if no barometric effect had occurred are shown below.

TABLE No. 11A.—*Buffalo, August 5, 1910.*

	Observed elevation of water surface.	Computed barometric effect.	Corrected elevation of water surface.
	feet.	*feet.*	*feet.*
1 a.m.	572.12	+.26	571.86
2	572.26	+.26	572.00
3	572.74	+.27	572.47
4	572.60	+.29	572.31
5	572.49	+.29	572.20
6	572.92	+.29	572.63
7	572.90	+.30	572.60
8	572.76	+.32	572.44
9	572.62	+.32	572.30
10	572.86	+.33	572.53
11 a.m.	572.67	+.35	572.32
N.	572.90	+.36	572.54
1 p.m.	573.12	+.38	572.74
2	573.18	+.36	572.82
3	573.12	+.35	572.77
4	573.13	+.34	572.79
5	573.30	+.34	572.96
6	573.48	+.33	573.15
7	573.28	+.34	572.94
8	573.28	+.32	572.96
9	572.96	+.30	572.66
10	572.68	+.29	572.39
11 p.m.	572.39	+.26	572.13
M.	572.49	+.25	572.24

The observed elevations are referred to mean sea-level.

The values of *L*, see equation (66), page 56, as recorded in the observation equations shown on page 58, in units of 0.01 foot, were obtained by subtraction of adjacent values of the corrected elevations shown above. The absolute term *L*, in each of the two observation equations, for the hours 3 and 4 a.m. and for the hours 5 and 6 a.m., is the corrected rise for two hours instead of one.

The observation equations were arranged in groups, one group for each day. The groups were studied separately as well as in combination with each other as a single set of equations for one whole solution.

EXAMPLES OF NORMAL EQUATIONS FOR WIND EFFECTS.

In each least-square solution for wind effects a set of normal equations was first formed in the usual way from the observation equations for each day. Then these sets of normal equations, one set for each day, were combined by addition to form the final set of normal equations for the whole solution.

The normal equations for August 5, 1910, in solution W25, formed from the observation equations for that date, as shown on page 58, are as follows:

$$\left.\begin{aligned} +252168C_p+134728C_a-21619&=0 \\ +134728C_p+260143C_a-12627&=0 \end{aligned}\right\} \tag{67}$$

The final normal equations for solution W25 for 22 days, formed by combining 22 such sets of normal equations as are shown in (67), are as follows:

$$\left.\begin{aligned} +2914495C_p+\ 588119C_a-205977&=0 \\ +\ 588119C_p+3343518C_a-178350&=0 \end{aligned}\right\} \tag{68}$$

This final set of normal equations for solution W25, (68), depends upon 470 observation equations, covering 500 hours, which formed parts of 22 days.

The solution of the final normal equations for solution W25, (68), gave the following values for the unknowns:

$$C_p=+0.0622\pm.0064 \qquad C_a=+0.0425\pm.0060$$

The probable errors shown were computed rigorously from these normal equations and from the residuals, v, of the 470 observation equations of solution W25.

THE FOUR FINAL WIND SOLUTIONS.

In this investigation, made by methods outlined on pages 6–8, several separate least-square solutions were usually made on the basis of each group of observational data. The form of each successive solution from one group of data was based upon all the information available up to the time that said form was adopted, including the information from earlier solutions based upon the same data.

The four final solutions for wind effects were those designated as solutions W25, W26, W20, and W29. Each of these solutions was the culminating one of a series of solutions based on the same group of observational data. The principal facts for these four solutions are shown in tabular form following:

TABLE No. 12.—*Principal facts for four final wind solutions.*

	Solution W25.	Solution W26.	Solution W20.	Solution W29.
Gage at which observations were made	Buffalo.	Buffalo.	Cleveland.	Cleveland.
No. of days of observation used in the solution	22	26	22	30
Total No. of hours of observation available	519	622	519	717
No. of hours covered by the observation equations finally used	500	614	508	669
No. of observation equations	470	583	490	630
Computed value of C_p	+.0622	+.0733	+.0237	+.0700
Computed probable error of C_p	±.0064	±.0106	±.0110	±.0122
Computed value of C_a	+.0425	−.0008	+.0358	+.0090
Computed probable error of C_a	±.0060	±.0095	±.0114	±.0126
Probable error of a single observation	±.107 ft.	±.085 ft.	±.058 ft.	±.052 ft.

The probable errors of C_p and C_a as shown were computed rigorously from the normal equations and the residuals of the observation equations.

The probable error of a single observation as shown above is the probable error, computed rigorously from the normal equations and the residuals, of the observed change in elevation of the water surface at the gage in any one hour.

As indicated on page 57, in connection with an exposition of the meaning of the quantities C_p and C_a, if C_p and C_a are equal there is no lag in the change of elevation of the water surface at the gage behind the change in the wind at the Weather Bureau station. If C_p is finite and C_a zero, the lag is 30 minutes, and if C_p is zero and C_a finite there is an anticipation of 30 minutes. An examination of the values of C_p and C_a in table No. 12 shows that solution W26 indicates a lag of 30 minutes, C_a being practically zero; that solutions W25 and W29 each indicate a lag of less than 30 minutes; and that solution W20 indicates an anticipation or negative lag of much less than 30 minutes. If one takes into account the degree of uncertainty indicated by the probable errors, as well as the smallness of the indicated lag in each of the four cases and the discrepancy between the four indications, the conclusion reached is that the actual lag is probably less than 30 minutes and is too small to be determined with reliability from the observations used. This conclusion that the lag is less than 30 minutes, and certainly too small to be determined from these observations, is confirmed by an examination of many details of the evidence.

In later portions of the investigation the lag in the wind effects was therefore assumed to be zero. All corrections for wind effect were computed on that basis. On that basis the C_x of the fundamental formula for wind effects is C_p+C_a.

Each solution gives one value for $C_x = C_p + C_a$.

The probable error of each value of C_x from one solution is, with sufficient accuracy for the present purpose, the square root of the sum of the squares of the separate probable errors of C_p and C_a from that solution.

Table No. 13 shows the four values of C_x and their probable errors, the weight assigned to each value of C_x, the weighted mean and its probable error, and the residuals of the four separate values from this weighted mean. The assigned weights are inversely proportional to the squares of the probable errors of C_x, corresponding to the assumption that all errors in computed values of C_x are of the accidental character. Unit weight corresponds to a probable error squared of 0.001.

TABLE No. 13.

	Probable error of C_x.	Assigned weight.	$C_x = C_p + C_a$.	Residual from weighted mean.
Solution W25, Buffalo.......	±0.0087	13.1	+0.105	−0.017
Solution W26, Buffalo.......	± .0142	4.9	+ .072	+ .016
Solution W20, Cleveland....	± .0158	4.0	+ .060	+ .028
Solution W29, Cleveland....	± .0175	3.2	+ .079	+ .009
		25.2	+0.088	

Sum of weights 25.2. Weighted mean +0.088. Probable error of weighted mean = ±0.006.

The value $+0.088 \pm 0.006$ for C_x is adopted as the best that can be derived from the present investigation. Accordingly, equation (51), expressing the total effect of the wind upon the elevation of the water surface at any gage, may now be written thus:

$$W = +0.088\left(\frac{h^{2.4}}{100}\right)\Sigma_x \tag{69}$$

In (69), W is expressed in units of 0.01 foot. h is the wind velocity at the gage as determined by observations at the nearest Weather Bureau station which makes such observations. Σ_x is a quantity dependent upon the location of the gage with reference to the lake, the shape of the lake, the depth in every part of the lake, and the direction of the wind. The meaning of Σ_x and the manner of computing it are fully stated on pages 43–54.

COMPUTATION OF WIND EFFECTS.

The wind effects at each hour at any gage might be computed by the use of formula (69) by making the proper substitutions for each hour of h and Σ_x. Such a substitution might be made systematically by a tabulation such as the first six columns of table No. 11 on page 59, supplemented by the last column of that table, which is headed W. Each value in that last

column is the W of formula (69), namely, the value in the sixth column of the table multiplied by $+0.088$.

The mean of the 24 values of W for August 5, 1910, at Buffalo gage, as shown in the last column of table No. 11, is $+0.258$ foot, which is therefore the daily wind effect for that day. The daily wind effects might be so computed.

Merely as devices to save time in the computation of daily wind effects, it was found to be advisable to make two changes in the method of computation which is indicated above. One change was to construct a table for each gage, giving the values of W in terms of the two arguments, wind velocity and wind direction. Note that in formula (69) there are only two variables in the second member, the wind velocity, h, and Σ_x, which is variable for a given gage as a function of the wind direction only. The other change was to make the actual values placed in this table $\frac{W}{24}$ instead of W itself. With the table before one and with the observed wind velocity and direction for each hour also before one, the tabular values, one for each hour, could be taken out very rapidly to the nearest thousandth of a foot. The sum of the 24 tabular values for any day was the daily wind effect. No division by 24 was necessary, as would otherwise have been the case, because each tabular value was itself the result of such a division.

ACCURACY OF COMPUTED BAROMETRIC EFFECTS.

Having set forth the theory and the methods of computation by which the formulæ and constants for computing barometric effects and wind effects have been determined, it is now proposed to set forth the main portions of the available evidence as to the accuracy and reliability with which the barometric effects and wind effects may be computed by the use of these formulæ and constants.

It is the purpose to set forth the evidence, first, in connection with the computed barometric effects; second, in connection with the computed wind effects; and, third, as to the overall accuracy attained when corrections are applied to each day's observations at each gage for both barometric and wind effects.

Let the evidence be considered here as to the accuracy of the computed barometric effects.

Table No. 6, page 32, shows each of the values of the various barometric constants B_{w0}, B_{w1}, etc., and its probable error as computed rigorously from the normal equations and the residuals of the least-square solution. These probable errors are a measure of the accuracy which is the best that can be obtained, provided the errors in the derived constants are all accidental in character. Assuming that the errors are all accidental, that is, that there are no systematic or constant errors affecting the final results, it is an even chance that the actual error in any constant is greater than or less than its

computed probable error as shown. For example, B_{n2} for Buffalo $= -11.69$, and its probable error is only ± 0.43. This means that the chances are even for and against the proposition that the true value of B_{n2} for Buffalo lies within 0.43 of -11.69. In other words, there is one chance in two that the value -11.69 is correct within one twenty-seventh of itself $\left(\frac{11.69}{0.43}=27\right)$.

If for each of the five stations this comparison be made between the largest constant for the station and its probable error, it will be found that in each case the probable error is as small as one-twentieth part, or 5 per cent of the value. The largest constant is selected because it is the one which tends to have most influence upon the computed barometric effect at the station.

On this basis the conclusion is that the computed barometric constants are subject to errors which stand one chance in two of being less than 5 per cent as large as the largest one of said constants at each station.

Note that the computed probable errors at each station are of approximately the same size for all constants, regardless of the size of the constant itself. The extreme uncertainty, judged by the probable errors, occurs in the constant B_{n3} for Harbor Beach, of which the value is $+0.47$, only 1.8 times its own probable error, ± 0.26. On the supposition that all the errors are accidental in character, there is less than one chance in four that this derived value, $+0.47$, for B_{n3} at Harbor Beach is entirely fictitious. For the few cases in table No. 6 like this one, in which there is a small chance that the constant is entirely fictitious, the main reliance must be placed on other tests than that furnished by the probable error when one is attempting to determine the accuracy and reliability.

If in making any least-square solution a large number of rejections is made of observations which show large residuals, a fictitious appearance of a high degree of accuracy may thereby be given to the remaining observations, which necessarily agree more closely with each other than did the original observations before any rejections were made. That danger has been guarded against in this investigation by the adoption of a cautious rejection limit and by a careful study of each rejected value to determine from external evidence if possible whether the value may properly be rejected. The external evidence has in a large percentage of cases been clearly in favor of the rejection. It is believed, therefore, that there is no danger that any fictitious accuracy has been introduced by the rejections.

THE REJECTION RULE.

The rejection rule has been that an observation shall be rejected if its residual is larger than five times the probable error of the observation, and that no other observations shall be rejected.

If all the errors were strictly accidental in character, this rule would reject less than one observation per thousand observations.

The number of rejections was much larger than one per thousand. As an example, in solution L2, the final barometric solution for Buffalo, 13 days of observation were rejected out of a total of 243 possible days in the months used in the solution. This is at the rate of 54 observations per thousand. In solution K2, the final barometric solution for Milwaukee, the rejections were at the rate of 53 per thousand.

The external evidence, obtained from a detailed study of the separate cases, is strong in indicating that as a rule, in the rejected cases, the observed change of elevation of the water surface on the day in question was abnormal and due to the first oscillation of a new seiche affecting the gage record at the station, or that it was abnormal because of extremely rapid and irregular changes in the barometric gradients over the lake, which departed widely from the conditions postulated in the approximate theory used in this investigation. Such rapid and irregular changes in barometric gradients were associated ordinarily with the passage of a powerful storm center, a low-pressure area, over the lake or near it. A discussion of the seiches will be found in a later part of this publication.

Throughout the investigation any observation which showed a residual larger than 3.5 times the probable error of the observation was considered as suspicious. In each such case a special study of the external evidence was made.

Early in the investigation it was found that such a residual was frequently preceded or followed by a residual of the opposite sign, which was also of considerable size. A study in detail of many such cases, using the evidence which was external to the least-square solution, showed that very frequently such a case was due to an abnormal effect, of one or the other of the two kinds mentioned above in connection with rejections, which occurred mainly within the limits of a single day. This effect made the observed elevation of the water surface for that one day either abnormally high or abnormally low. In the first case there was an apparently abnormal rise of the water surface on one day, followed on the next day by an abnormal fall. In the second case there was apparently an abnormal fall on one day, followed by an abnormal rise on the next. After many such cases had been examined in detail the following rule as to combining successive observation equations was adopted and used thereafter throughout the investigation.

RULE FOR COMBINING OBSERVATION EQUATIONS.

> Whenever any observation equation has a residual larger than 3.5 times the probable error of a single observation, and a residual for an equation immediately preceding or immediately following in time is of the opposite sign, the two observation equations shall be combined to form one equation, provided the residual for the new combined observation equation will be less than 3.5 times the probable error of a single observation.

Such a procedure rejects the observed elevation of the water surface on one day and treats a two-day interval as the basis of observation instead of a

one-day interval, so far as the one combined equation is concerned. It retains all of the observed facts as to changes in barometric gradients and uses them in the combined equation.

As already stated, page 25, the combined observation equation was formed by adding the two separate equations term by term.

If the residuals were due entirely to accidental errors, only 18 equations per thousand would have residuals larger than the adopted suspicion limit, -3.5 times the probable error. Much less than 18 equations per thousand should, if the residuals were due entirely to accidental errors, be subject to the combination rule. For if all errors were accidental, only a few of the suspicious residuals would be preceded or followed immediately by a residual of opposite sign big enough to bring the combined residual within the suspicion limit.

In table No. 6 it is shown that the number of days of observation used in each solution exceeded the number of equations from 6 per cent (at Mackinaw) to 22 per cent (at Buffalo). In a few cases two or more days were used in one observation equation, because no record of the elevation of the water surface was obtainable from the gage for one or more days. In each such case an observation equation was ordinarily used covering the interval during which the gage record was missing. After allowing for such cases, without a definite count, it appears that the number of combinations made under the above-stated rule for combining observation equations varied from about 50 to about 200 per thousand. This is far in excess of the number of such combinations which would occur if the residuals were entirely due to accidental error. The external evidence supported the combinations, of one or the other of the two kinds mentioned in connection with rejections, as being justified by something peculiar to the one abnormal day.

DISCREPANCIES BETWEEN PAIRS OF VALUES.

Each least-square solution gives two values for C_w and two values for C_n, as noted on page 23. The discrepancy between the two values of each pair is a test of the accuracy of the adopted value of C_w or C_n. Table No. 14 gives the discrepancies of that character arising from the final solutions for barometric effects which gave the adopted barometric constants shown in table No. 6, page 32. The equations (36) to (39), referred to in the first column, are shown on page 23.

According to the laws of probability, the discrepancy between two values should be upon an average about three times the probable error of their mean. In table No. 14, 5 of the 10 discrepancies are less than 10 per cent of the larger of the two C's for that station. The extreme case is the discrepancy of 1.94 between the two values of C_n for Cleveland, which is 29 per cent of the adopted value of C_n at that station. Table No. 14 indicates, therefore, that the errors in the computed values of C_w and C_n are probably less than 3 per cent of the larger of said values at each station.

Table No. 14.

	Buffalo.	Cleveland.	Milwaukee.	Harbor Beach.	Mackinaw.
Designation of solution	L2	M2	K2	N1	O1
C_w from equation (36)....	+ 3.74	+1.73	−4.70	+7.91	−2.78
C_w from equation (37)....	+ 5.70	+2.52	−5.24	+5.97	−2.29
Discrepancy between above two values............	1.96	.79	.54	1.94	.49
Adopted mean of above two values................	+ 4.72	+2.12	−4.97	+6.94	−2.54
C_n from equation (38)....	−15.65	+7.77	+8.33	− .51	−4.32
C_n from equation (39)....	−15.60	+5.83	+8.56	− .37	−4.44
Discrepancy between above two values............	.05	1.94	.23	.14	.12
Adopted mean of above two values................	−15.62	+6.80	+8.44	− .44	−4.38

Each least-square solution gives two values for the lag in each barometric effect, E–W and N–S, as noted on page 22. The discrepancy in each pair between the two computed values of the lag is a test of the accuracy of the adopted value of the lag. Table No. 15 gives the discrepancies of that character arising from the final solutions for barometric effects. The lags were computed by use of table No. 2, as indicated on page 22.

In table No. 15, of the 10 possible discrepancies, 3 did not develop, since there was a failure to get a determination of the lag, as indicated in the footnote.

Of the remaining 7 discrepancies, 5 range from 2.0 hours to 7.0 hours, and 2 are 0.2 hour and 0.1 hour, for the lag in C_n at Milwaukee and Buffalo, respectively.

Note that by comparison with table No. 7, page 33, the two pairs of determinations of lag which gave the very small discrepancies noted at Milwaukee and Buffalo were for very large values of C_n, +8.44 and −15.62, respectively; also, that in the case in which both possible determinations of lag failed for C_n at Harbor Beach, the value of C_n involved is very small, −0.44.

In general, then, it appears that the lag in the barometric effects is apparently determined within less than an hour for the largest effects, is subject to a possible uncertainty of several hours for the very smallest effects, and for the effects of intermediate size the chances are about even that the error in the lag is only one or two hours.

When one notes how slowly the barometric effects change from hour to hour it appears that such determinations of lag represent a rather high degree of accuracy in the determination of barometric effects. At Buffalo, where C_n is −15.62, the largest value of the kind encountered in this investigation, the maximum change in the N–S barometric effect in one hour found in the computations was 0.07 foot. The discrepancy between the

lag determinations in this case, 0.1 hour, therefore corresponded to a discrepancy in elevation of the water surface of less than 0.01 foot.

TABLE No. 15.

	Buffalo.	Cleveland.	Milwaukee.	Harbor Beach.	Mackinaw.
Designation of solution	L2	M2	K2	N1	O1
Lag in C_w from $\frac{B_{w_2}}{B_{w_1}}$.....	−0.5 hr	+5.6 hr	+8.2 hr	+4.4 hr	−5.0 hr
Lag in C_w from $\frac{B_{w_3}}{B_{w_0}}$.....	*	+2.1 hr	+6.2 hr	+8.4 hr	+2.0 hr
Discrepancy between above two values.....	....	3.5 hr	2.0 hr	4.0 hr	7.0 hr
Adopted mean of above two values...........	−1 hr	+4 hr	+7 hr	+6 hr	+2 hr
Lag in C_n from $\frac{B_{n_2}}{B_{n_1}}$.....	+3.3 hr	+0.4 hr	+4.3 hr	*	1.1 hr
Lag in C_n from $\frac{B_{n_3}}{B_{n_0}}$.....	+3.4 hr	+4.3 hr	+4.1 hr	*	*
Discrepancy between above two values.....	0.1 hr	3.9 hr	0.2 hr	....	....
Adopted mean of above two values...........	+3 hr	+2 hr	+4 hr	+6 hr†	−1 hr

For the purposes of this table the words hour and hours are designated as hr.

Values marked with minus signs are negative lags or anticipations.

*In these cases no determination of the lag was possible, because the two values of which the ratio should serve to determine the lag were of opposite sign.

†This value, not determinable from the observations, was assumed to be +6 hours, the same as the lag as determined for the E–W effects at this station.

STUDY OF PROPORTIONALITY FACTORS.

Additional evidence as to the accuracy and reliability of the computed barometric effects is afforded by a study of the proportionality factors P_w and P_n at the various stations.

The formulæ (16) and (17), page 16, for barometric effects were derived upon the assumption that the water of the lake remained continuously in equilibrium under the influence of gravity and the barometric pressure. The proportionality factors P_w and P_n were then introduced (see pages 16–18) to take account of the modifications which would probably be produced by friction and by inertia. It was recognized that such modifications would probably be dependent to a considerable extent upon the configuration of the shores and bottom of the lake and might to a considerable extent be peculiar to each gage location. The modified equations for barometric effects which were the basis of this investigation are shown as (18) and (19) on page 17.

Table No. 16 shows grouped together for convenient inspection, (1) the values of C_w and C_n derived from the final solutions (see table No. 7, page 33), (2) the values of R_w and R_n computed as indicated in formula (17), page 16, for each gage station, and (3) the various values of P_w and P_n derived by substitution of the values of C_w, C_n, R_w, and R_n in equation (19).

TABLE No. 16.

	Buffalo.	Cleveland.	Milwaukee.	Harbor Beach.	Mackinaw.
Designation of solution	L2	M2	K2	N1	O1
C_w	+ 4.72	+2.12	−4.97	+6.94	− 2.54
R_w	+ 3.55	− .75	−5.14	+3.10	− .12
P_w	+ 1.33	−2.83	+ .97	+2.24	+21.17
C_n	−15.62	+6.80	+8.44	−0.44	− 4.38
R_n	− 1.72	+1.39	+3.20	+1.38	− 3.03
P_n	+ 9.08	+4.89	+2.64	− .32	+ 1.45

If the actual barometric effects were those corresponding to continuous equilibrium of the water under the influence of gravity and barometric pressure, and if all assumptions made in deriving formulæ (16) and (17) were true, the computed values of P_w and P_n would be 1.00 at every station within the limits fixed by the errors of observation. The wide range of values of P_w and P_n in table No. 16, from −2.83 to +21.17, is ample evidence that decided modifications of the barometric effects are produced by friction and inertia, and that there are probably appreciable errors in the assumptions used as a basis for the computations. The wide range also indicates that the modifications and errors are peculiar to each station, not common to them all.

The extreme value +21.17 for P_w at Mackinaw is probably due to two causes. The gage at Mackinaw is very slightly to the westward of the center of gravity of the area of Lake Michigan–Huron (see plate 2). Hence, the value of R_w for that gage is very small, −0.12. The Strait of Mackinac probably acts as a throttle, to a certain extent, between Lake Michigan and Lake Huron. It probably tends to delay the delivery of water back and forth between the two lakes, which must occur under the influence of barometric changes if the two lakes are to act as one. Any delay in the delivery of the water through the Strait of Mackinac tends to make the two lakes act temporarily as separate lakes. If the water surface at Mackinaw gage acted as if it were a part of the surface of Lake Huron alone, the L_w of such a formula as (17) for that gage should be measured from the center of gravity of Lake Huron alone and would be much greater than the L_w which was actually measured from the center of gravity of the combined lakes. In that case, R_w would be much larger than −0.12, while P_w would be much smaller than +21.17. In other words, if the oscillation of the water of Lake

Huron under the influence of barometric changes is about its own center of gravity as a nodal point, the rise and fall at Mackinaw gage would be much greater under fluctuation of E–W barometric gradients than if the oscillation were about the center of gravity of the combined lakes—Lake Michigan–Huron—as assumed in the computations. The extremely large value of P_w at the Mackinaw gage, $+21.17$, is believed, therefore, to be a strong indication of a throttling effect in the Strait of Mackinac.

The negative value of P_n at Harbor Beach, -0.32, is believed to be due to a peculiar effect of the Strait of Mackinac arising from the fact that it is the only connection between Lake Michigan and Lake Huron, and that said connection is almost at the extreme northern end of each lake. The fact that B_{n0} and B_{n1} at Harbor Beach (see table No. 6, page 32) have apparently abnormal negative signs, while B_{n2} and B_{n3} have the positive signs, which would be normal according to the derivation of the formula used in this investigation, is also believed to be due to the same cause.

No adequate explanation is offered for the apparently abnormal value of P_w at Cleveland, -2.83. It is surmised that it is related in some way to the following three facts: (1) the center of gravity of Lake Erie, which normally should be the nodal point for barometric effects, is not far to the eastward of Cleveland; (2) the nodal line for east or west winds on Lake Erie is to the westward of Cleveland; and (3) there is a decided bend in the south shore of Lake Erie at Cleveland. Note that the minus sign for P_w at Cleveland indicates that the actual nodal point for barometric effects on Lake Erie is to the westward of Cleveland.

The remaining seven values of P_w and P_n on which no special comments have been made are all positive, their mean value is $+3.23$, and the largest two values are $+9.08$ and $+4.89$ for P_n at Buffalo and Cleveland, respectively. Inertia tends strongly to make P_w and P_n greater than $+1.00$. Other evidence, set forth later in this publication, indicates that inertia effects upon the elevation of the water surface are probably much greater on Lake Erie than on Lake Michigan–Huron. The effects of errors in assumption No. 2 (see page 16) are probably greater for N–S effects on Lake Erie than in any other case in this investigation, since the ratio of the distance from point 5 to point 7 (see plate 2) to the maximum extent of Lake Erie in the N–S direction is greater than any other similar ratio involved in assumption No. 2. Note that for the reason stated on page 16 the departure of the facts from assumption No. 2 is likely to be such as to make the values of P_w and P_n considerably greater than $+1.00$. For the reasons indicated briefly in this paragraph, it is believed that the seven values of P_w and P_n here commented upon have in them no indications of inaccuracy of observation or computation or unreliability of the theory on which the computations were made.

Taking into account all ten values of P_w and P_n, the comments made upon them above, and other consideration of details not here set forth, the general conclusions reached are as follows:

(1) The barometric effects as computed have in general, barring certain classes of exceptional cases referred to in (4), the degree of accuracy indicated by the computed probable errors.

(2) There are decided modifications of the barometric effects, due to the configuration of the shores and bottom of the lake, which modifications are peculiar to each gage station and in general increase the magnitude of the effects.

(3) The errors in assumption No. 2 probably contribute to making P_w and P_n greater than $+1.00$.

(4) At times, when the changes in barometric gradients are unusually rapid and irregular, such as the times when a well-developed low-pressure area is passing over or near a lake, the errors in the computed barometric effects are probably abnormally large. At such a time the shape of the wave produced on the lake surface by barometric changes is unusual, and therefore one may expect the modifications in it due to inertia and dependent upon the configuration of the shores and bottom to be unusual.

CONCLUSIONS ON ACCURACY OF COMPUTED BAROMETRIC EFFECTS.

A comparison of each final least-square solution, such as L2 at Buffalo, with earlier least-square solutions, furnishes very valuable and instructive evidence as to the accuracy and reliability of the conclusions from the final solutions. But such evidence is made up of many details and many considerations of such a character that they can not be briefly presented. Hence, it must suffice here to state that such evidence furnished strong corroboration of the foregoing discussion of the errors in the computed barometric effects.

The general conclusions, based upon all the evidence available, which have been reached in regard to the computed barometric effects at the five stations Buffalo, Cleveland, Milwaukee, Harbor Beach, and Mackinaw are:

(1) The errors of the computed daily barometric effects are probably less than 5 per cent of said effects in about one-half of all days.

(2) On a small percentage of exceptional days, at times when the barometric gradients are changing rapidly and irregularly, usually when a well-developed low-pressure area is over or near the lake, the computed barometric effects are subject to abnormally large errors, much greater than 5 per cent. These exceptional days are usually solitary, not in groups.

ACCURACY OF COMPUTED WIND EFFECTS.

As shown on page 63, the final value adopted for the constant C_x of the fundamental formula for wind effects—(51) on page 39— was $+0.088$, which gave as the definite numerical formula for wind effects (69) on page 63.

What is the accuracy of the value $+0.088$? This value is a weighted mean of four separate values from four separate least-square solutions, as shown in table No. 13, page 63. Its probable error is ± 0.006, as there

shown, computed rigorously from the normal equations and the residuals of the four solutions. If all errors affecting the value +0.088 were of the accidental class, the chances would therefore be even that said value is correct within $\frac{1}{15}$ part $\left(\frac{0.006}{0.088}=\frac{1}{15}\right)$.

The four separate least-square solutions are, as indicated in table No. 12, page 62, based on four separate and independent sets of observed data, elevations of the water surface at two different gages—Buffalo and Cleveland —and including 2,291 hours of observation scattered over 100 days. If any systematic or constant errors affected the computed values of C_x from the separate solutions, they would be apt to appear in the comparison of those four values which is shown in table No. 13. If the errors are all of the accidental class, each of such residuals as are shown in the last column of that table should have an even chance of being less than the probable error written in the same line in that table. Note that the residual for solution W29, +0.009, is much less than the corresponding probable error, ±0.0175; that the residual for solution W26, +0.016, is very slightly greater than the corresponding probable error, ±0.0142; and that the residual which is largest in proportion to its corresponding error is that for solution W25, −0.017, which is 1.9 times the corresponding probable error, ±0.0087. According to the laws of probability, such a residual, 1.9 times the probable error, should occur on an average once in five times. It appears, then, that the agreement of the four values of C_x from the four solutions is so close as to furnish no indication of systematic or constant errors. The degree of agreement indicates that all errors are of the accidental class, and therefore that the degree of accuracy of the value +0.088 is that indicated by its computed probable error, ±0.006.

The same rules as to rejections and combinations were used in the computation of wind effects as have already been stated in connection with the computation of barometric effects (see pages 65–66).

The same considerations which led there to the conclusion that there was no danger that a fictitious accuracy had been imparted to the computed barometric effects as a result of the rejections and combinations also led to the same conclusion with respect to the computed wind effects.

As shown in table No. 12, of the total of 2,377 hours of observation available, 2,291 hours were used. That is, only 86 hours, or less than 4 per cent, were rejected.

The total number of separate observation equations was 2,173—118 less, or 5 per cent less, than the total number of hours used. That is, the combinations of equations which were made were such as to combine 5 per cent of the hours with adjacent hours instead of using them in separate independent equations.

In addition to the rejections and combinations made under the regular rules, referred to above, in solution W29 at Cleveland, a few other combinations and rejections were made on the basis of evidence external to the

solution. These were based mainly upon evidence which indicated that when there was an apparent shift of the wind direction between the directions N and NE or between the directions S and SW the water surface at Cleveland in frequent cases did not respond in the normal manner. There were two surmises as to the reason, one connected with the possible behavior of the water itself and one a suspicion as to the wind record.

Plate 2 shows that with a shift of the wind from N to NE or from S to SW there is a very large shift in the position of the nodal line on Lake Erie. This means that for a slight change in direction of the wind, only 45°, there must be a transfer of an unusually large amount of water over an unusually long distance before the new steady régime is established. Accordingly, at the time of said shifts it is possible that there are decidedly unusual temporary fluctuations in elevations of water surface at various points due to inertia effects in the unusual currents which must occur at those times. Cleveland, lying between the two locations of the nodal line for S and for SW winds, may be especially subject to such temporary fluctuations of elevation of water surface.

The other surmise arose from two considerations:

First, in talking with officials of the Weather Bureau in both Chicago and Washington the idea was several times advanced that at particular stations there might be errors in the recorded direction of the wind at the station, due to the immediate surroundings of the station, such that the recorded wind direction of the station might not be truly representative of the wind direction in a large region around the station. For example, it is possible that for certain hours a wind may be recorded as NE at the Weather Bureau station at Cleveland though the actual wind blowing at that time over the western half of Lake Erie and influencing the water surface might be a north wind rather than a northeast wind.

Second, for certain days in which there seemed to be reason to suspect the validity of the Cleveland wind directions in the respect just noted, a comparison was made of the wind directions recorded at Cleveland and those recorded at Buffalo and other stations. These comparisons in some cases indicated fickle fluctuations of wind between N and NE as recorded at Cleveland when the record at other stations showed no reason to apprehend such fluctuations. It was therefore surmised that possibly there might be a tendency to station error in the wind direction at Cleveland, affecting especially the directions N, NE, S, and SW. This is the only case in this investigation in which there appears to be any reason to suspect an appreciable station error in either wind direction or wind velocity.

The total number of rejections and combinations made on the unusual basis indicated above was moderate. Table No. 12 shows that the total number of rejections in solution W29 at Cleveland was only 7 per cent and the total of combinations was only 6 per cent, whereas the corresponding percentages for all four solutions combined were 4 and 5, respectively.

The external evidence seemed to be ample and convincing that all these

rejections and combinations should be made. Aside from the special cases in solution W29 at Cleveland indicated above, the rejections and combinations appear to be due, as a rule, to seiches, and especially to the first and largest oscillation of a new seiche which has just been started by an unusually vigorous impulse given by a change in wind or by a change in barometric gradients.

Aside from the four final solutions for wind effects of which the results are given in table No. 13, page 63, and which fixed the adopted value of C_x, three other wind solutions for different gage stations were also made, namely, one each at Milwaukee, Harbor Beach, and Mackinaw. These three gave determinations of C_x which have such large probable errors that the values were not used in fixing the adopted value of C_x. These solutions were not made with all the refinements of the final solutions. Nevertheless, they furnish a valuable check on the correctness of the theory as to wind effects which has been used in the investigation.

In table No. 17, showing the results of these three solutions, the assigned weights are on the same basis as the weights shown in table No. 13.

TABLE No. 17.

	Probable error of C_x.	Assigned weight.	$C_x = C_p + C_a$.	Residual from adopted final value of C_x, viz., +.088.
Solution W16, Milwaukee...	±0.134	0.06	−0.229	+0.317
Solution W17, Harbor Beach	± .081	.15	− .218	+ .306
Solution W18, Mackinaw...	± .245	.02	+ .246	− .158
Sum of weights........		= .23		

Note that the sum of the weights for these three solutions is less than 0.01 as great as the sum of the weights for the four final solutions, as shown in table No. 13.

Note that the residuals for these three solutions are not large enough to prove that systematic or constant errors are present. One of the three residuals is less than the corresponding probable error, and in the extreme case, solution W17, the residual is 3.8 times the corresponding probable error. According to the laws of probability, a residual 3.8 times the corresponding error should occur once in about 100 times.

If these three solutions, with their proper weights, as shown above, were used with the four solutions of table No. 13 to fix the final adopted value of C_x, that value would be +0.086, differing only 0.002 from that actually adopted.

Consider the values of Σ_x which are shown in table No. 10, page 53, and note the contrast between the values for the two lakes concerned. The maximum value for the Lake Erie stations is 8.32 and the minimum value is 1.72.

On the other hand, the maximum value at any of the three Lake Michigan–Huron stations is 0.95, but little more than one-half of the minimum on Lake Erie. Of the 24 values at the Lake Michigan–Huron stations, 6 are less than one-tenth as large as the minimum (1.72) at Lake Erie stations. Roughly speaking, then, the Lake Michigan–Huron effects, which must be proportional to Σ_x if the theory used in this investigation is correct, must be less than one-tenth as large as the wind effects on Lake Erie, at the stations under consideration. Such minute wind effects as those postulated on Lake Michigan–Huron must be very difficult to detect by the observation of water-surface elevations, masked as they necessarily are by the much larger barometric effects and by seiches.

Table No. 18 shows how small the wind effects are at the three Lake Michigan–Huron stations, both in absolute units and in comparison with the wind effects at the two stations on Lake Erie. The values are as computed from formula (69), page 63.

TABLE No. 18—*Maximum value of the wind effect at any hour within the limits of this investigation at various gage stations.*

	Date and hour.	Wind velocity in miles per hour.	Wind direction.	Wind effect, feet.
Buffalo.........	Oct. 27, 1910, 10 a.m	53	SW	1.003
Cleveland......	Oct. 27, 1910, 7 a.m.	41	W	.186
Milwaukee......	Sept. 6, 1911, midnight	31	E	.032
Harbor Beach...	July 24, 1911, 3 p.m.	35	W	.034
Mackinaw......	Aug. 25, 1910, 6 p.m.	33	W	.016

When it is realized that the maximum wind effect at any one of the three stations on Lake Michigan–Huron during the months covered by the investigation was only 0.034 foot even for a single hour, as shown in the table, and that during more than one-half of the time the wind effect is certainly less than 0.010 foot, then it is clear that the large probable errors in table No. 17 are due to this cause. So, also, are the abnormal minus signs on two of the computed values of C_x. The wind effects were too small to show through the mask of barometric effects and seiches.

To what extent is the accuracy of the computed wind effects reduced by uncertainty as to the true value of the exponent of h in formula (51), page 39? It has already been stated, pages 54–55, that no theory is known which is deemed adequate to fix the value of this exponent, that it has in this investigation been derived from the observations, that the observations indicate 2.4 to be the most probable value, and that the belief has been reached that whatever error exists in this adopted value has a very slight influence on the accuracy of the computed wind effects. It is appropriate to indicate somewhat more definitely at this point the basis of the belief stated in the last part of the preceding sentence.

At Cleveland, six least-square solutions for wind effects were made, in which the assumed values of the exponent were 2.0, 2.2, 2.3, and 2.4. Comparisons were made between these solutions, in pairs, to determine as far as feasible the effect of the assumed change of exponent between the two solutions of each pair. The change of exponent from 2.0 to 2.2 reduced Σv^2 by 3 per cent, and other kinds of internal evidence showed clearly that 2.2 is nearer the truth than 2.0. The expression Σv^2 is used to indicate the sum of the squares of the residuals in a solution. It is evident that the closer the approach to the truth in all assumptions the smaller will be Σv^2. The change of the assumed exponent from 2.2 to 2.3 reduced Σv^2 only 1 per cent, and from 2.3 to 2.4 reduced it again only 1 per cent. In these last two steps the other evidence than that given by Σv^2 was not clear.

At Buffalo, seven least-square solutions for wind effects were made, in which the assumed values of the exponent were 2.0, 2.3, 2.4, and 2.5. There was decisive evidence that the change from 2.0 to 2.3 was an approach to the truth. But among the six of the seven solutions which were based on the assumed values 2.3, 2.4, and 2.5 for the exponent, the apparent change in Σv^2 due to change of exponent over this range of 0.2 was only 1 per cent or less. Other evidence was not clear in favor of any choice between 2.4 and 2.5.

Hence, the conclusions reached from a consideration of the evidence as a whole are (*a*) that the most probable value of the exponent is 2.4, (*b*) that the true value may be as low as 2.3 or as high as 2.5, and (*c*) that the error in the assumed value 2.4 probably produces a very small otherwise-avoidable error in the final computed daily wind effects, provided the one value of the exponent is carried consistently throughout the whole computation.

The fact that a change of 0.1 in the exponent either way from 2.4 produces a change of only 1 per cent or less in Σv^2 is the main basis for conclusion (*c*) above, which is applicable primarily to daily wind effects. Such a daily effect is the mean of 24 hourly effects, which usually involve winds of a considerable range of velocity and usually a change of direction. It is conceded that the computed wind effect for the hour of maximum velocity may be appreciably in error on account of the error in the exponent, but the other values for lighter winds during the day will tend normally to have much less error from this cause, and for the extremely light winds the error tends to be reversed in sign. The net result is a daily wind effect based on 24 hourly values having little error due to the cause in question.

The adopted exponent, 2.4, depends on deductions from observations, not primarily on theory, and is believed to be as good an approximation to the truth as is needed for the prime purpose of this investigation.

The evidence as set forth on pages 72–77, mainly from the four final least-square solutions for wind effects, is abundantly corroborated by the many earlier least-square solutions made in series on the general plan indicated on pages 6–8. There were in all 29 least-square solutions devoted entirely to a study of wind effects, and 37 other solutions in which the wind effects were studied in conjunction with other matters.

ACCURACY OF CORRECTED ELEVATIONS OF WATER SURFACE.

There has been presented in some detail in the preceding pages the manner in which the barometric effects and wind effects have been computed for five gage stations on the Great Lakes. Some of the evidence has been set forth as to the accuracy of these computed values. It is important to secure as decisive tests as are feasible of the conclusions which have been reached as to the accuracy and reliability of the corrections for barometric effects and wind effects. With that end in view, the tabulations and comments of the following pages dealing with observed and corrected elevations of the water surface for each day, for each interval of five days, and for intervals of one month and one season are set forth in turn for each of the five gage stations and for each of the two lakes considered as a unit. It is believed that these tabulations and comments furnish the most decisive tests of accuracy and reliability that are feasible within the time limits of this investigation and of its exposition in print.

In tables Nos. 19–23, which follow in order for Buffalo, Cleveland, Milwaukee, Harbor Beach, and Mackinaw, the barometric corrections and the wind corrections as shown were computed as indicated on pages 36–39, 63–64. These are corrections, and are therefore of the opposite sign from barometric effects and wind effects. The observed elevation for each day as shown is the mean of 24 hourly elevations of the water surface as observed at the gage specified. The corrections for barometric effect and wind effect for the day being applied to the observed elevation gives the corrected elevation as shown in the next column. This corrected elevation is the best value of the mean elevation of the whole lake surface that can be obtained from that gage for that day, as distinguished from the elevation of any part of the surface. The main purpose of these tables, though not the only one, is to show the degree of accuracy and reliability with which the mean elevation of the whole lake surface is determined from day to day, and thereby the fluctuation in total content of the lake determined.

All elevations in these tables are referred to mean sea-level.

In the column headed "5-day observed mean," each value is in nearly all cases the mean of five daily observed elevations. The exceptions are of two kinds. First, in some cases the group includes six values, and the mean is therefore for six days. For example, the six days, July 26–31, 1910, are grouped together in order to have the beginning of the next 5-day group on the first day of the month. Second, the group is sometimes for less than five days when there has been an interruption in the gage record. For example, there was no observed elevation available for August 11, 1910. Hence, the group normally covering the days August 11–15 contains but four values, and the mean in the column marked "5-day observed mean" is for four values only.

The column marked "5-day corrected mean" bears the same relation to the corrected elevations as the column marked "5-day observed mean" does to the observed elevations. The explanation made in the preceding paragraph for groups containing six values or less than five values also applies to the corrected elevations and 5-day corrected means.

In the column "Corrected elevation," an occasional value is inclosed in parentheses and the corresponding residual is also inclosed in parentheses. Each corrected elevation contained in a parenthesis, such, for example, as 2.36 on June 2, 1910, at Buffalo, is one which has been identified by a definite criterion, based on the preceding investigation, as being an abnormal disturbed value which should be rejected in taking means. It is so rejected in this tabulation. The criterion will be stated later in the proper context.

The residuals of the daily observed elevations from the 5-day observed means, and the corresponding residuals of daily corrected elevations from the 5-day corrected means, as shown side by side in the last two columns of the table, furnish an instructive indication of the relative accuracy of the observed and corrected elevations.

The preceding general explanations also apply to tables Nos. 20 to 23.

MEAN ELEVATIONS OF LAKE ERIE.

In tables Nos. 19 and 20 there are given elevations of the water surface of Lake Erie as observed at the Buffalo gage and at the Cleveland gage. Before this investigation had been made, the best available approximation to the mean elevation of the whole surface of Lake Erie on a given day would have been assumed to be the mean of these two observed elevations at the two gages. Also, there are given in tables Nos. 19 and 20 the corrected elevations from those gages. Each such corrected elevation is a value for the elevation of the mean surface of Lake Erie after the corrections have been applied for disturbances at each gage by barometric pressures and by winds. The mean of these two corrected elevations, one for the Buffalo gage and one for the Cleveland gage, is recognized from this investigation to be the best approximation, on any given day, to the mean elevation of the whole surface of Lake Erie for that day.

Table No. 24 serves to place in juxtaposition these two mean values of the mean elevation of the whole surface of Lake Erie, one without corrections and the other with corrections for barometric effects and wind effects. The table serves to enable one to compare the two sets of values, study their accuracy by means of the 5-day means and the residuals, and so make progress in testing the over-all accuracy and reliability of the barometric corrections and wind corrections.

In table No. 24 the observed elevation for any day was obtained by merely taking a mean of the two observed elevations for that day as recorded in table No. 19 for Buffalo and table No. 20 for Cleveland.

Table No. 19—*Observed and corrected elevations of water surface at the Buffalo Gage on Lake Erie.*

Date.	Barometric correction.	Wind correction.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
June 1...	−0.31	−0.11	2.94	2.52			−0.38	−0.02
2...	− .16	− .06	2.58	(2.36)			− .02	(+ .14)
3...	− .04	+ .01	2.57	2.54	2.56	2.50	− .01	− .04
4...	+ .11	+ .02	2.27	2.40			+ .29	+ .10
5...	+ .09	.00	2.46	2.55			+ .10	− .05
6...	− .10	− .06	2.65	2.49			− .09	+ .03
7...	− .24	− .02	2.77	2.51			− .21	+ .01
8...	− .13	− .03	2.69	2.53	2.56	2.52	− .13	− .01
9...	+ .13	+ .02	2.43	2.58			+ .13	− .06
10...	+ .22	+ .03	2.26	2.51			+ .30	+ .01
11...	+ .18	+ .02	2.29	2.49			+ .30	− .05
12...	− .28	− .14	2.78	2.36			− .19	+ .08
13...	− .22	− .03	2.63	2.38	2.59	2.44	− .04	+ .06
14...	− .15	− .04	2.63	2.44			− .04	00
15...	− .08	− .02	2.63	2.53			− .04	−. 09
16...	− .03	.00	2.54	2.51			+ .05	− .01
17...	− .17	− .02	2.61	2.42			− .02	+ .08
18...	− .16	− .02	2.68	2.50	2.59	2.50	− .09	.00
19...	− .05	.00	2.57	2.52			+ .02	− .02
20...	.00	− .01	2.55	2.54			+ .04	− .04
21...	+ .05	.00	2.55	2.60			− .10	− .05
22...	+ .02	− .02	2.54	2.54			− .09	+ .01
23...	.00	− .01	2.56	2.55	2.45	2.55	− .11	.00
24...	+ .23	+ .04	2.20	2.47			+ .25	+ .08
25...	+ .21	.00	2.39	2.60			+ .06	− .05
26...	− .01	.00	2.41	2.40			+ .12	+ .04
27...	− .20	− .03	2.59	2.36			− .06	+ .08
28...	− .08	− .02	2.60	2.50	2.53	2.44	− .07	− .06
29...	− .09	− .02	2.55	2.44			− .02	.00
30...	− .03	− .01	2.52	2.48			+ .01	− .04
July 1...	− .14	− .03	2.58	2.41			− .24	− .08
2...	− .10	− .01	2.48	2.37			− .14	− .04
3...	.00	− .05	2.52	(2.47)	2.34	2.33	− .18	(− .14)
4...	+ .19	+ .04	2.02	2.25			+ .32	+ .08
5...	+ .17	+ .01	2.12	2.30			+ .22	+ .03
6...	.00	.00	2.30	2.30			+ .11	− .06
7...	− .19	− .01	2.44	2.24			− .03	.00
8...	− .14	− .01	2.39	2.24	2.41	2.24	+ .02	.00
9...	− .21	− .04	2.46	2.21			− .05	+ .03
10...	− .21	− .04	2.47	2.22			− .06	+ .02

TABLE No. 19—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means.	
							Obs.	Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
July 11...	−0.09	−0.01	2.42	2.32			+0.03	+0.05
12...	− .02	− .02	2.42	2.38			+ .03	− .01
13...	− .17	− .04	2.62	2.41	2.45	2.37	− .17	− .04
14...	+ .03	.00	2.38	2.41			+ .07	− .04
15...	− .06	− .01	2.39	2.32			+ .06	+ .05
16...	− .03	+ .02	2.18	2.17			− .04	+ .03
17...	+ .11	+ .03	1.95	2.09			+ .19	+ .11
18...	+ .17	+ .03	2.03	2.23	2.14	2.20	+ .11	− .03
19...	+ .06	.00	2.23	2.29			− .09	− .09
20...	− .09	.00	2.29	2.20			− .15	.00
21...	− .26	− .05	2.39	2.08			+ .05	+ .08
22...	− .24	− .05	2.47	2.18			− .03	− .02
23...	− .06	− .01	2.28	2.21	2.44	2.16	+ .16	− .05
24...	− .15	− .07	2.42	2.20			+ .02	− .04
25...	− .39	− .12	2.64	2.13			− .20	+ .03
26...	− .18	− .05	2.46	2.23			− .04	+ .04
27...	− .11	− .03	2.49	2.35			− .07	− .08
28...	− .07	− .01	2.35	2.27	2.42	2.27	+ .07	.00
29...	− .10	− .02	2.38	2.26			+ .04	+ .01
30...	− .20	− .03	2.45	2.22			− .03	+ .05
31...	− .05	.00	2.36	2.31			+ .06	− .04
Aug. 1...	− .06	− .04	2.35	2.25			+ .13	.00
2...	.00	.00	2.28	2.28			+ .20	− .03
3...	− .03	.00	2.22	2.19	2.48	2.25	+ .26	+ .06
4...	− .30	− .12	2.69	2.27			− .21	− .02
5...	− .31	− .26	2.84	2.27			− .36	− .02
6...	− .21	− .06	2.45	2.18			− .20	+ .01
7...	+ .06	.00	2.13	2.19			+ .12	+ .00
8...	+ .10	.00	2.17	2.27	2.25	2.19	+ .08	− .08
9...	+ .01	− .01	2.18	2.18			+ .07	+ .01
10...	− .14	− .03	2.32	2.15			− .07	+ .04
11...	.00	− .01	....	....			....	....
12...	+ .05	.00	2.01	2.06			+ .05	.00
13...	− .04	.00	2.08	2.04	2.06	2.06	− .02	+ .02
14...	− .03	.00	2.10	2.07			− .04	− .01
15...	+ .02	.00	2.04	2.06			+ .02	.00
16...	+ .11	.00	1.98	2.09			+ .06	− .04
17...	+ .07	− .01	2.01	2.07			+ .03	− .02
18...	− .10	− .02	2.16	2.04	2.04	2.05	− .12	+ .01
19...	+ .03	− .01	2.05	2.07			− .01	− .02
20...	.00	− .01	2.01	2.00			+ .03	+ .05

TABLE No. 19—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Aug. 21...	−0.04	0.00	1.97	1.93			+0.16	−0.04
22...	− .11	− .01	2.11	1.99			+ .02	− .10
23...	− .17	− .03	2.04	1.84	2.13	1.89	+ .09	+ .05
24...	− .19	− .06	2.10	1.85			+ .03	+ .04
25...	− .45	− .14	2.45	1.86			− .32	+ .03
26...	− .21	− .06	2.30	(2.03)			− .46	(− .09)
27...	− .03	− .01	2.03	1.99			− .19	− .05
28...	+ .12	+ .01	1.81	1.94			+ .03	.00
29...	+ .30	+ .05	1.28	(1.63)	1.84	1.94	+ .56	(+ .31)
30...	+ .21	.00	1.73	1.94			+ .11	.00
31...	+ .02	− .03	1.88	1.87			− .04	+ .07
Sept. 1...	+ .09	.00	1.78	1.87			+ .12	+ .07
2...	+ .16	+ .02	1.91	(2.09)			− .01	(− .13)
3...	− .13	− .05	....	....	1.90	1.94	....	....
4...	+ .05	.00	....	....			....	....
5...	+ .02	− .02	2.01	2.01			− .11	− .07
6...	− .25	− .13	2.43	(2.05)			− .35	(− .17)
7...	− .17	− .03	2.09	1.89			− .01	− .01
8...	− .08	− .01	1.99	1.90	2.08	1.88	+ .09	− .02
9...	− .20	− .01	2.09	1.88			− .01	.00
10...	+ .04	+ .01	1.80	1.85			+ .28	+ .03
11...	+ .03	.00	1.85	1.88			− .05	− .06
12...	− .04	− .02	1.92	1.86			− .12	− .04
13...	+ .04	+ .01	1.72	1.77	1.80	1.82	+ .08	+ .05
14...	+ .06	+ .01	1.68	1.75			+ .12	+ .07
15...	+ .03	.00	1.82	1.85			− .02	− .03
16...	+ .05	.00	1.71	1.76			+ .02	+ .01
17...	− .07	.00	1.80	1.73			− .07	+ .04
18...	− .03	.00	1.83	1.80	1.73	1.77	− .10	− .03
19...	+ .13	+ .02	1.44	(1.59)			+ .29	(+ .18)
20...	− .06	− .02	1.87	1.79			− .14	− .02
21...	+ .07	.00	1.72	1.79			− .09	− .03
22...	+ .28	+ .02	1.27	(1.57)			+ .36	(+ .19)
23...	+ .10	.00	1.66	1.76	1.63	1.76	− .03	.00
24...	+ .21	+ .03	1.52	1.76			+ .11	.00
25...	− .17	− .05	1.96	1.74			− .33	+ .02
26...	+ .08	+ .02	1.52	1.62			+ .24	.00
27...	− .18	− .08	2.09	(1.83)			− .33	(− .21)
28...	− .11	− .01	1.82	1.70	1.76	1.62	− .06	− .08
29...	+ .03	.00	1.57	1.60			+ .19	+ .02
30 ..	− .20	− .03	1.80	1.57			− .04	+ .05

TABLE No. 19—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Oct. 1...	−0.40	−0.20	2.42	(1.82)			−0.57	(−0.24)
2...	+ .04	.00	1.55	1.59			+ .30	− .01
3...	− .01	− .02	1.62	1.59	1.85	1.58	+ .23	− .01
4...	− .28	− .10	1.94	1.56			− .09	+ .02
5...	− .26	− .03	1.74	(1.45)			+ .11	(+ .13)
6...	− .13	.00	1.86	1.73			+ .01	+ .03
7...	+ .09	+ .01	1.66	1.76			+ .21	.00
8...	− .06	− .01	1.83	1.76	1.87	1.76	+ .04	.00
9...	− .29	− .02	2.08	1.77			− .21	− .01
10...	− .08	− .06	1.92	1.78			− .05	− .02
11...	− .25	− .16	2.31	(1.90)			− .46	(− .14)
12...	+ .16	+ .02	1.53	1.71			+ .32	+ .05
13...	+ .21	.00	1.50	1.71	1.85	1.76	+ .35	+ .05
14...	− .10	− .02	1.93	1.81			− .08	− .05
15...	− .14	− .05	1.99	1.80			− .14	− .04
16...	− .23	− .04	1.97	1.70			− .24	.00
17...	+ .09	.00	1.65	1.74			+ .08	− .04
18...	− .06	− .01	1.76	1.69	1.73	1.70	− .03	+ .01
19...	− .07	− .02	1.78	1.69			− .05	+ .01
20...	+ .18	+ .03	1.47	1.68			+ .26	+ .02
21...	+ .16	+ .02	1.36	(1.54)			+ .72	(+ .19)
22...	− .48	− .22	3.11	(2.41)			−1.03	(− .68)
23...	− .26	− .04	2.04	1.74	2.08	1.73	+ .04	− .01
24...	− .21	− .08	2.05	1.76			+ .03	− .03
25...	− .14	− .03	1.85	1.68			+ .23	+ .05
26...	− .09	.00	1.69	1.60			+ .37	+ .07
27...	− .35	− .22	2.63	(2.06)			− .57	(− .39)
28...	− .21	− .04	....	....	2.06	1.67	....	....
29...	− .08	.00	1.82	1.74			+ .24	− .07
30...	− .10	− .11	2.34	(2.13)			− .28	(− .46)
31...	− .23	− .06	1.80	(1.51)			+ .26	(+ .16)

TABLE No. 20—*Observed and corrected elevations of water surface at the Cleveland Gage on Lake Erie.*

Date.	Barometric correction.	Wind correction.	Observed elevation 570+.	Corrected elevation. 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
June 1...	−0.08	−0.01	2.57	2.64			+0.08	0.00
2...	+ .03	.00	2.60	2.63			+ .05	+ .01
3...	− .07	− .01	2.71	2.63	2.65	2.64	− .06	+ .01
4...	− .01	.00	2.70	2.69			− .05	− .05
5...	− .06	.00	2.67	2.61			− .02	+ .03
6...	− .02	.00	2.62	2.60			+ .01	+ .02
7...	+ .02	− .02	2.59	2.59			+ .04	+ .03
8...	+ .05	.00	2.58	2.63	2.63	2.62	+ .05	− .01
9...	− .02	+ .01	2.66	2.65			− .03	− .03
10...	− .09	+ .02	2.72	2.65			− .09	− .03
11...	− .11	+ .02	2.75	2.66			− .11	+ .01
12...	+ .10	− .01	2.59	2.68			+ .05	− .01
13...	+ .09	.00	2.63	2.72	2.64	2.67	+ .01	− .05
14...	+ .04	.00	2.62	2.66			+ .02	+ .01
15...	+ .01	.00	2.63	2.64			+ .01	+ .03
16...	− .01	.00	2.64	2.63			− .01	.00
17...	+ .04	− .01	2.62	2.65			+ .01	− .02
18...	+ .02	− .02	2.61	2.61	2.63	2.63	+ .02	+ .02
19...	.00	.00	2.65	2.65			− .02	− .02
20...	− .02	.00	2.62	2.60			+ .01	+ .03
21...	− .03	.00	2.61	2.58			− .02	− .01
22...	− .02	.00	2.59	2.57			.00	.00
23...	− .02	.00	2.58	2.56	2.59	2.57	+ .01	+ .01
24...	− .09	.00	....	....			....	....
25...	− .04	+ .02	....	....			....	....
26...	+ .05	.00	....	....			....	....
27...	+ .09	.00	....	....			....	....
28...	.00	.00	....	....	2.43	2.43	....	....
29...	+ .01	.00	2.42	2.43			+ .01	.00
30...	− .01	.00	2.44	2.43			− .01	.00
July 1...	+ .02	.00	2.38	2.40			+ .05	+ .01
2...	+ .04	.00	2.40	2.44			+ .03	− .03
3...	− .01	.00	2.40	2.39	2.43	2.41	+ .03	+ .02
4...	− .14	+ .01	2.56	2.43			− .13	− .02
5...	− .02	+ .01	2.41	2.40			+ .02	+ .01
6...	+ .06	.00	2.36	2.42			− .02	+ .01
7...	+ .07	.00	2.36	2.43			− .02	.00
8...	+ .09	.00	2.37	2.46	2.34	2.43	− .03	− .03
9...	+ .13	.00	2.31	2.44			+ .03	− .01
10...	+ .10	− .01	2.31	2.40			+ .03	+ .03

TABLE No. 20—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means.	
							Obs.	Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
July 11...	+0.07	0.00	....	....			....	....
12...	+ .06	+ .01	2.30	2.37			+0.12	+0.08
13...	+ .03	.00	2.38	2.41	2.42	2.45	+ .04	+ .04
14...	.00	.00	2.49	2.49			− .07	− .04
15...	+ .02	.00	2.51	2.53			− .09	− .08
16...	− .01	.00	2.58	(2.57)			− .03	(− .17)
17...	− .08	+ .01	2.64	(2.57)			− .09	(− .17)
18...	− .11	− .01	2.54	2.42	2.55	2.40	+ .01	− .02
19...	− .06	.00	2.43	2.37			+ .12	+ .03
20...	+ .05	.00	....	....			....	....
21...	+ .13	.00	2.28	2.41			+ .05	− .02
22...	+ .10	.00	2.29	2.39			+ .04	.00
23...	+ .05	.00	2.33	2.38	2.33	2.39	.00	+ .01
24...	+ .12	+ .03	....	....			....	....
25...	+ .15	− .02	2.41	(2.54)			− .08	(− .15)
26...	+ .06	.00	2.43	2.49			− .03	− .04
27...	+ .06	.00	2.33	2.39			+ .07	+ .06
28...	.00	.00	2.42	2.42	2.40	2.45	− .02	+ .03
29...	+ .06	.00	2.37	2.43			+ .03	+ .02
30...	+ .03	.00	2.47	2.50			− .07	− .05
31...	− .05	.00	2.40	(2.35)			.00	(+ .10)
Aug. 1...	+ .05	.00	2.37	2.42			− .02	− .01
2...	+ .02	.00	2.40	2.42			− .05	− .01
3...	+ .06	.00	2.37	2.43	2.35	2.41	− .02	− .02
4...	+ .10	− .02	2.35	2.43			.00	− .02
5...	+ .10	− .04	2.27	2.33			+ .08	+ .08
6...	+ .09	− .01	2.28	2.36			− .03	− .07
7...	+ .01	.00	2.28	2.29			− .03	.00
8...	− .01	.00	2.23	2.22	2.25	2.29	+ .02	+ .07
9...	+ .09	.00	2.18	2.27			+ .07	+ .02
10...	+ .07	.00	2.26	2.33			− .01	− .04
11...	.00	.00	2.26	2.26			.00	+ .02
12...	+ .02	− .01	2.29	2.30			− .03	− .02
13...	+ .09	.00	....	....	2.26	2.28	....	....
14...	+ .08	.00	....	....			....	....
15...	+ .05	.00	2.24	2.29			+ .02	− .01
16...	+ .03	.00	2.25	2.28			− .01	+ .02
17...	+ .08	+ .01	2.21	2.30			+ .03	.00
18...	+ .09	.00	2.24	2.33	2.24	2.30	.00	− .03
19...	− .01	− .01	2.29	2.27			− .05	+ .03
20...	+ .07	.00	2.23	2.30			+ .01	.00

Table No. 20—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means. Obs.	Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Aug. 21...	+0.12	0.00	2.17	2.29			−0.09	−0.03
22...	+ .16	+ .01	2.08	2.25			.00	+ .01
23...	+ .16	+ .01	2.09	2.26	2.08	2.26	− .01	.00
24...	+ .20	+ .02	2.03	2.25			+ .05	+ .01
25...	+ .24	.00	2.02	2.26			+ .06	.00
26...	+ .05	− .02	2.12	2.15			.00	− .03
27...	+ .02	.00	2.04	2.06			+ .08	+ .06
28...	− .03	.00	2.11	2.08			+ .01	+ .04
29...	− .11	+ .03	2.26	2.18	2.12	2.12	−. 14	− .06
30...	+ .03	+ .02	2.06	2.11			+ .06	+ .01
31...	.00	.00	2.11	2.11			+ .01	+ .01
Sept. 1...	− .11	+ .01	2.19	2.09			− .06	+ .05
2...	− .03	.00	2.14	2.11			− .01	+ .03
3...	+ .09	.00	2.05	2.14	2.13	2.14	+ .08	.00
4...	− .01	.00	2.20	2.19			− .07	− .05
5...	+ .06	+ .01	2.08	2.15			+ .05	− .01
6...	+ .12	− .02	2.03	2.13			+ .08	+ .01
7...	+ .02	.00	2.10	2.12			+ .01	+ .02
8...	+ .08	.00	2.12	2.20	2.11	2.14	− .01	− .06
9...	− .03	− .02	2.21	2.16			− .10	− .02
10...	− .03	.00	2.10	2.07			+ .01	+ .07
11...	+ .01	.00	2.07	2.08			− .01	− .07
12...	+ .03	.00	2.02	2.05			+ .04	− .04
13...	− .12	− .01	2.13	2.00	2.06	2.01	− .07	+ .01
14...	− .12	− .01	2.11	1.98			− .05	+ .03
15...	− .06	.00	1.98	1.92			+ .08	+ .09
16...	− .05	.00	....	....			....	....
17...	+ .04	.00	....	....			....	....
18...	.00	.00	1.93	1.93	1.97	1.96	+ .04	+ .03
19...	− .05	.00	2.06	2.01			− .09	− .05
20...	+ .02	.00	1.93	1.95			+ .04	+ .01
21...	− .06	− .01	1.99	1.92			− .04	+ .01
22...	− .07	.00	2.05	1.98			− .10	− .05
23...	+ .02	.00	1.91	1.93	1.95	1.93	+ .04	.00
24...	.00	+ .01	1.90	1.91			+ .05	+ .02
25...	+ .02	.00	1.89	1.91			+ .06	+ .02
26...	− .05	.00	....	....			....	....
27...	+ .06	.00	1.86	1.92			.00	− .02
28 ..	.00	.00	1.89	1.89	1.86	1.90	− .03	+ .01
29...	− .01	.00	1.90	1.89			− .04	+ .01
30...	+ .13	.00	1.79	1.92			+ .07	− .02

TABLE No. 20—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Oct. 1...	+0.13	−0.05	1.79	1.87			−0.03	+0.02
2...	+ .01	.00	1.86	1.87			− .10	+ .02
3...	+ .13	+ .02	1.74	1.89	1.76	1.89	+ .02	.00
4...	+ .21	+ .03	1.64	1.88			+ .12	+ .01
5...	+ .16	+ .02	1.78	1.96			− .02	− .07
6...	− .03	− .04	2.28	(2.21)			− .18	(− .20)
7...	− .10	− .01	2.13	2.02			− .03	− .01
8...	.00	.00	2.03	2.03	2.10	2.01	+ .07	− .02
9...	− .01	− .02	2.07	2.04			+ .03	− .03
10...	− .03	.00	1.97	1.94			+ .13	+ .07
11...	+ .05	− .01	1.79	1.83			+ .12	+ .05
12...	− .13	+ .01	2.07	1.95			− .16	− .07
13...	− .08	+ .01	....	....	1.91	1.88	....	....
14...	− .02	.00	1.88	1.86			+ .03	+.02
15...	− .02	.00	1.90	1.88			+ .01	.00
16...	− .02	− .01	1.89	1.86			+ .07	+ .05
17...	− .07	.00	1.98	1.91			− .02	.00
18...	+ .02	.00	1.94	1.96	1.96	1.91	+ .02	− .05
19...	+ .03	+ .01	....	....			....	....
20...	− .15	.00	2.05	1.90			− .09	+ .01
21...	− .08	.00	2.09	2.01			− .26	− .06
22...	+ .14	− .04	1.56	(1.66)			+ .27	(+ .29)
23...	+ .05	− .02	1.92	1.95	1.83	1.95	− .09	.00
24...	+ .08	+ .02	1.64	(1.74)			+ .19	(+ .21)
25...	− .03	− .02	1.93	1.88			− .10	+ .07
26...	+ .04	.00	1.85	1.89			− .13	− .03
27...	+ .12	− .06	1.74	1.80			− .02	+ .06
28...	+ .07	− .06	1.85	1.86	1.72	1.86	− .13	.00
29...	+ .02	− .04	1.94	1.92			− .22	− .06
30...	+ .20	+ .03	1.33	(1.56)			+ .39	(+ .30)
31...	+ .17	.00	1.64	1.81			+ .08	+ .05

TABLE No. 21—*Observed and corrected elevations of water surface at the Milwaukee Gage on Lake Michigan–Huron.*

Date.	Baro-metric cor-rection.	Wind cor-rection.	Observed eleva-tion 579+.	Cor-rected eleva-tion 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means.	
							Obs.	Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
June 1...	+0.20	0.00	0.79	0.99			+0.21	−0.01
2...	+ .03	.00	.96	.99			+ .04	− .01
3...	− .11	.00	1.06	.95	1.00	.98	− .06	+ .03
4...	− .16	.00	1.13	.97			− .13	+ .01
5...	− .06	.00	1.04	.98			− .04	.00
6...	− .12	.00	1.11	.99			− .04	+ .05
7...	− .06	.00	1.03	.97			+ .04	+ .07
8...	− .03	.00	1.10	1.07	1.07	1.04	− .03	− .03
9...	− .03	.00	1.12	1.09			− .05	− .05
10...	+ .05	.00	1.01	1.06			+ .06	− .02
11...	+ .06	.00	.96	1.02			+ .01	+ .05
12...	+ .06	.00	.94	1.00			+ .03	+ .07
13...	+ .14	.00	.97	1.11	.97	1.07	.00	− .04
14...	+ .13	.00	.98	1.11			− .01	− .04
15...	+ .09	.00	1.02	1.11			− .05	− .04
16...	+ .04	.00	1.07	1.11			− .01	− .04
17...	− .07	.00	1.11	1.04			− .05	+ .03
18...	+ .02	.00	1.04	1.06	1.06	1.07	+ .02	+ .01
19...	+ .07	.00	1.00	1.07			+ .06	.00
20...	+ .03	.00	1.06	1.09			.00	− .02
21...	+ .03	.00	1.05	1.08			+ .09	+ .01
22...	+ .06	.00	1.00	1.06			+ .14	+ .03
23...	− .08	.00	1.16	1.08	1.14	1.09	− .02	+ .01
24...	− .17	.00	1.27	1.10			− .13	− .01
25...	− .12	.00	1.23	1.11			− .09	− .02
26...	− .01	.00	1.13	1.12			− .06	− .02
27...	+ .10	.00	.99	1.09			+ .08	+ .01
28...	+ .07	.00	.99	1.06	1.07	1.10	+ .08	+ .04
29...	− .04	.00	1.13	1.09			− .06	+ .01
30...	+ .02	.00	1.13	1.15			− .06	− .05
July 1...	+ .08	.00	1.00	1.08			+ .01	− .01
2...	+ .15	.00	.93	1.08			+ .08	− .01
3...	+ .06	.00	1.00	1.06	1.01	1.07	+ .01	+ .01
4...	− .02	.00	1.07	1.05			− .06	+ .02
5...	+ .01	.00	1.06	1.07			− .05	.00
6...	+ .05	.00	1.05	1.10			+ .01	+ .03
7...	− .01	.00	1.15	1.14			− .09	− .01
8...	+ .01	.00	1.16	1.17	1.06	1.13	− .10	− .04
9...	+ .10	.00	1.00	1.10			+ .06	+ .03
10...	+ .18	.00	.94	(1.12)			+ .12	(+ .01)

TABLE No. 21—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means. Obs.	Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
July 11...	+0.16	0.00	0.85	1.01			+0.02	−0.01
12...	+ .14	.00	.81	.95			+ .06	+ .05
13...	+ .13	.00	.87	1.00	.87	1.00	.00	.00
14...	+ .10	.00	.89	.99			− .02	+ .01
15...	+ .12	.00	.92	1.04			− .05	− .04
16...	+ .04	.00	.95	.99			− .04	+ .01
17...	+ .13	.00	.88	1.01			+ .03	− .01
18...	+ .15	.00	.89	1.04	.91	1.00	+ .02	− .04
19...	+ .05	.00	.97	1.02			− .06	− .02
20...	+ .12	.00	.84	.96			+ .07	+ .04
21...	+ .12	.00	.76	.88			− .06	− .02
22...	+ .16	.00	.77	.93			− .07	− .07
23...	+ .02	.00	.82	.84	.70	.86	− .12	+ .02
24...	+ .21	.00	.65	.86			+ .05	.00
25...	+ .30	.00	.48	.78			+ .22	+ .08
26...	+ .12	.00	.74	.86			+ .06	− .01
27...	+ .07	.00	.81	.88			− .01	− .03
28...	+ .03	.00	.84	.87			− .04	− .02
29...	+ .06	.00	.80	.86	.80	.85	.00	− .01
30...	+ .04	.00	.78	.82			+ .02	+ .03
31...	− .02	.00	.83	.81			− .03	+ .04
Aug. 1...	− .03	.00	.91	.88			− .03	− .02
2...	+ .06	.00	.83	.89			+ .05	− .03
3...	− .06	.00	.86	.80	.88	.86	+ .02	+ .06
4...	− .06	.00	.93	.87			− .05	− .01
5...	.00	.00	.85	.85			+ .03	+ .01
6...	− .04	.00	.87	.83			+ .02	+ .06
7...	− .07	.00	.95	.88			− .06	+ .01
8...	+ .12	.00	.80	.92	.89	.89	+ .09	− .03
9...	+ .10	.00	.77	.87			+ .12	+ .02
10...	− .09	.00	1.06	.97			− .17	− .08
11...	− .05	.00	1.06	1.01			− .10	− .03
12...	− .05	.00	.95	(.90)			+ .01	(+ .08)
13...	+ .01	.00	.96	.97	.96	.98	.00	+ .01
14...	+ .04	.00	.95	.99			+ .01	− .01
15...	+ .07	.00	.86	.93			+ .10	+ .05
16...	+ .12	.00	.84	.96			+ .01	− .03
17...	+ .10	.00	.88	.98			− .03	− .05
18...	+ .08	.00	.83	.91	.85	.93	+ .02	+ .02
19...	+ .06	.00	.86	.92			− .01	+ .01
20...	+ .04	.00	.85	.89			.00	+ .04

TABLE No. 21—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means. Obs.	Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Aug. 21...	+0.08	0.00	0.73	0.81			+0.06	+0.01
22...	+ .01	.00	.84	.85			− .05	− .03
23...	+ .04	.00	.77	.81	.79	.82	+ .02	+ .01
24...	− .03	.00	.83	.80			− .04	+ .02
25...	+ .02	.00	.80	.82			− .01	.00
26...	+ .03	.00	.76	.79			− .02	− .05
27...	− .02	.00	.77	.75			− .03	− .01
28...	+ .08	.00	.67	.75	.74	.74	+ .07	− .01
29...	+ .04	.00	.70	.74			+ .04	.00
30...	− .09	.00	.75	.66			− .01	+ .08
31...	− .06	.00	.79	.73			− .05	+ .01
Sept. 1...	+ .06	.00	.67	.73			+ .05	+ .01
2...	+ .10	.00	.66	.76			+ .06	− .02
3...	+ .06	.00	.67	.73	.72	.74	+ .05	+ .01
4...	− .04	.00	.78	.74			− .06	.00
5...	− .08	.00	.80	.72			− .08	+ .02
6...	− .08	.00	.92	.84			− .01	+ .01
7...	− .21	.00	1.08	.87			− .17	− .02
8...	− .14	.00	.99	.85	.91	.85	− .08	.00
9...	+ .02	.00	.82	.84			+ .09	+ .01
10...	+ .10	.00	.74	.84			+ .17	+ .01
11...	+ .08	.00	.71	.79			− .01	− .06
12...	+ .06	.00	.73	.79			− .03	− .06
13...	− .04	.00	.76	.72	.70	.73	− .06	+ .01
14...	− .07	.00	.70	.63			.00	+ .10
15...	+ .11	.00	.61	.72			+ .09	+ .01
16...	+ .11	.00	.59	.70			+ .11	+ .02
17...	− .08	.00	.83	.75			− .13	− .03
18...	− .02	.00	.79	.77	.70	.72	− .09	− .05
19...	+ .10	.00	.60	.70			+ .10	+ .02
20...	.00	.00	.70	.70			.00	+ .02
21...	− .09	.00	.78	.69			− .04	+ .03
22...	+ .08	.00	.64	.72			+ .10	.00
23...	+ .06	.00	.63	.69	.74	.72	+ .11	+ .03
24...	− .05	.00	.82	.77			− .08	− .05
25...	− .06	.00	.81	.75			− .07	− .03
26...	.00	.00	.65	.65			+ .07	+ .03
27...	+ .02	.00	.67	.69			+ .05	− .01
28...	− .07	.00	.75	.68	.72	.68	− .03	.00
29...	− .06	.00	.86	(.80)			− .14	(− .12)
30...	.00	.00	.68	.68			+ .04	.00

TABLE No. 22—*Observed and corrected elevations of water surface at the Harbor-Beach Gage on Lake Huron.*

Date.	Barometric correction.	Wind correction.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means.	
							Obs.	Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
June 1...	−0.11	0.00	1.02	0.91			−0.11	+0.01
2...	− .03	.00	.90	.87			+ .01	+ .05
3...	+ .05	.00	.86	.91	0.91	0.92	+ .05	+ .01
4...	+ .09	.00	.86	.95			+ .05	− .03
5...	+ .04	.00	.93	.97			− .02	− .05
6...	+ .05	.00	.94	.99			− .02	− .02
7...	.00	.00	.94	.94			− .02	+ .03
8...	+ .02	.00	.95	.97	.92	.97	− .03	.00
9...	+ .13	.00	.86	.99			+ .06	− .02
10...	+ .07	.00	.91	.98			+ .01	− .01
11...	+ .04	.00	.97	1.01			+ .09	.00
12...	.00	.00	1.06	1.06			.00	− .05
13...	− .11	.00	1.10	.99	1.06	1.01	− .04	+ .02
14...	− .11	.00	1.11	1.00			− .05	+ .01
15...	− .09	.00	1.07	.98			− .01	+ .03
16...	− .05	.00	....	....			....	....
17...	− .01	.00	....	....			....	....
18...	− .06	.00	1.06	1.00	1.07	1.00	+ .01	.00
19...	− .04	.00	1.06	1.02			+ .01	− .02
20...	− .09	.00	1.08	.99			− .01	+ .01
21...	− .02	.00	1.03	1.01			− .07	+ .01
22...	− .02	.00	1.05	1.03			− .09	− .01
23...	.00	.00	1.04	1.04	.96	1.02	− .08	− .02
24...	+ .10	.00	.82	(.92)			+ .14	(+ .10)
25...	+ .10	.00	.88	.98			+ .08	+ .04
26...	+ .07	.00	.89	.96			+ .08	+ .01
27...	− .04	.00	.98	.94			− .01	+ .03
28...	− .12	.00	1.14	1.02	.97	.97	− .17	− .05
29...	.00	.00	.98	.98			− .01	− .01
30...	+ .08	.00	.86	.94			+ .11	+ .03
July 1...	+ .04	.00	.95	.99			+ .02	.00
2...	− .01	.00	1.00	.99			− .03	.00
3...	− .05	.00	1.02	.97	.97	.99	− .05	+ .02
4...	+ .02	.00	.98	1.00			− .01	− .01
5...	+ .09	.00	.91	1.00			+ .06	− .01
6...	− .07	.00	1.06	.99			− .12	− .01
7...	− .01	.00	1.00	.99			− .06	− .01
8...	+ .10	.00	.86	.96	.94	.98	+ .08	+ .02
9...	+ .13	.00	.85	.98			+ .09	.00
10...	+ .03	.00	.94	.97			.00	+ .01

Table No. 22—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
July 11...	+0.02	0.00	1.01	1.03			−0.02	−0.04
12...	− .06	.00	1.08	1.02			− .09	− .03
13...	− .02	.00	.99	.97	0.99	0.99	.00	+ .02
14...	+ .02	.00	.98	1.00			+ .01	− .01
15...	+ .04	.00	.88	.92			+ .11	+ .07
16...	− .02	.00	.96	.94			− .04	− .01
17...	− .06	.00	1.03	.97			− .11	− .04
18...	+ .02	.00	.89	.91	.92	.93	+ .03	+ .02
19...	+ .07	.00	.80	.87			+ .12	+ .06
20...	+ .03	.00	.91	.94			+ .01	− .01
21...	− .02	.00	.94	.92			− .01	+ .01
22...	− .04	.00	.98	.94			− .05	− .01
23...	+ .08	.00	.85	.93	.93	.93	+ .08	.00
24...	+ .02	− .02	.68	(.68)			+ .25	(+ .25)
25...	− .21	.00	1.22	(1.01)			− .29	(− .08)
26...	− .06	.00	.97	.91			− .12	− .02
27...	+ .02	.00	.86	.88			− .01	+ .01
28...	+ .12	.00	.77	.89	.85	.89	+ .08	.00
29...	+ .03	.00	.84	.87			+ .01	+ .02
30...	+ .04	.00	.84	.88			+ .01	+ .01
31...	+ .08	.00	.83	.91			+ .02	− .02
Aug. 1...	+ .12	.00	.71	.83			+ .09	+ .02
2...	+ .03	.00	.84	.87			− .04	− .02
3...	− .02	.00	.89	.87	.80	.85	− .09	− .02
4...	+ .06	.00	.77	.83			+ .03	+ .02
5...	+ .04	.00	.81	.85			− .01	.00
6...	+ .01	.00	.85	.86			− .02	− .04
7...	+ .10	.00	.72	.82			+ .11	.00
8...	− .07	.00	.83	.76	.83	.82	.00	+ .06
9...	− .05	.00	.90	.85			− .07	− .03
10...	− .02	.00	.85	.83			− .02	− .01
11...	− .06	.00	.88	.82			− .03	+ .03
12...	+ .01	.00	.86	.87			− .01	− .02
13...	+ .05	.00	.80	.85	.85	.85	+ .05	.00
14...	+ .03	.00	.84	.87			+ .01	− .02
15...	− .04	.00	.89	.85			− .04	.00
16...	− .04	.00	.94	.90			+ .04	+ .02
17...	− .07	.00	.97	.90			+ .01	+ .02
18...	− .11	.00	1.08	.97	.98	.92	− .10	− .05
19...	− .09	.00	1.01	.92			− .03	.00
20...	.00	.00	.91	.91			+ .07	+ .01

TABLE No. 22—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means.	
							Obs.	Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Aug. 21...	+0.08	0.00	0.83	0.91			+0.05	−0.04
22...	− .04	.00	.95	.91			− .07	− .04
23...	− .10	.00	.98	.88	0.88	0.87	− .10	− .01
24...	.00	.00	.82	.82			+ .06	+ .05
25...	+ .02	.00	.81	.83			+ .07	+ .04
26...	+ .04	.00	.80	.84			− .03	− .05
27...	+ .11	.00	.72	.83			+ .05	− .04
28...	− .04	.00	.84	.80	.77	.79	− .07	− .01
29...	− .08	.00	.99	(.91)			− .22	(− .12)
30...	+ .07	.00	.69	.76			+ .08	+ .03
31...	+ .12	.00	.59	.71			+ .18	+ .08
Sept. 1...	+ .02	.00	.65	.67			.00	− .02
2...	+ .01	.00	.71	.72			− .06	− .07
3...	− .13	.00	.81	.68	.65	.65	− .16	− .03
4...	.00	.00	.60	.60			+ .05	+ .05
5...	+ .09	.00	.49	.58			+ .16	+ .07
6...	− .05	.00	.66	.61			− .10	− .01
7...	+ .06	.00	.54	.60			+ .02	.00
8...	+ .07	.00	.44	(.51)	.56	.60	+ .12	(+ .09)
9...	+ .05	.00	.56	.61			.00	− .01
10...	− .01	.00	.60	.59			− .04	+ .01
11...	+ .05	.00	.57	.62			+ .03	+ .02
12...	− .07	.00	.84	(.77)			− .24	(− .13)
13...	+ .05	.00	.65	.70	.60	.64	− .05	− .06
14...	+ .21	.00	.43	.64			+ .17	.00
15...	+ .10	.00	.49	.59			+ .11	+ .05
16...	− .03	.00	.67	.64			− .08	− .03
17...	+ .07	.00	.52	.59			+ .07	+ .02
18...	+ .17	.00	.43	.60	.59	.61	+ .16	+ .01
19...	− .07	.00	.70	.63			− .11	− .02
20...	− .03	.00	.64	.61			− .05	.00
21...	+ .06	.00	.50	.56			+ .03	+ .03
22...	+ .06	.00	.57	.63			− .04	− .04
23...	+ .13	.00	.49	.62	.53	.59	+ .04	− .03
24...	− .02	.00	.58	.56			− .05	+ .03
25...	− .05	.00	.49	(.44)			+ .04	(+ .15)
26...	.00	.00	.57	.57			− .03	− .01
27...	+ .02	.00	.51	.53			+ .03	+ .03
28...	+ .03	.00	.57	.60	.54	.56	− .03	− .04
29...	.00	.00	.45	(.45)			+ .09	(+ .11)
30...	− .02	.00	.58	.56			− .04	.00

TABLE No. 23—*Observed and corrected elevations of water surface at Mackinaw Gage on Lake Michigan–Huron.*

Date.	Barometric correction.	Wind correction.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
June 1...	−0.04	0.00	0.95	0.91			−0.06	−0.02
2...	+ .05	.00	.87	.92			+ .02	− .03
3...	+ .02	.00	.85	.87	0.89	0.89	+ .04	+ .02
4...	+ .01	.00	.87	.88			+ .02	+ .01
5...	− .02	.00	.90	.88			− .01	+ .01
6...	+ .03	.00	.86	.89			+ .08	+ .02
7...	+ .02	.00	.91	.93			+ .03	− .02
8...	− .02	.00	.92	.90	.94	.91	+ .02	+ .01
9...	− .09	.00	.97	.88			− .03	+ .03
10...	− .10	.00	1.03	.93			− .09	− .02
11...	− .07	.00	1.06	.99			− .03	+ .03
12...	− .01	.00	1.06	1.05			− .03	− .03
13...	.00	.00	1.01	1.01	1.03	1.02	+ .02	+ .01
14...	+ .01	.00	1.01	1.02			+ .02	.00
15...	.00	.00	1.01	1.01			+ .02	+ .01
16...	− .01	.00	1.02	1.01			.00	+ .03
17...	+ .05	.00	1.02	1.07			.00	− .03
18...	+ .02	.00	1.01	1.03	1.02	1.04	+ .01	+ .01
19...	− .01	.00	1.04	1.03			− .02	+ .01
20...	+ .05	.00	1.00	1.05			+ .02	− .01
21...	− .02	.00	1.06	1.04			− .08	− .02
22...	− .02	.00	1.07	1.05			− .09	− .03
23...	+ .03	.00	.84	(.87)	.98	1.02	+ .14	(+ .15)
24...	.00	.00	.94	(.94)			+ .04	(+ .08)
25...	.00	.00	.98	.98			.00	+ .04
26...	− .06	.00	1.14	(1.08)			− .12	(− .10)
27...	− .02	.00	1.02	1.00			.00	− .02
28...	+ .05	.00	.94	.99	1.02	.98	+ .08	− .01
29...	+ .01	.00	.94	.95			+ .08	+ .03
30...	− .08	.00	1.04	.96			− .02	+ .02
July 1...	− .07	.00	1.04	.97			− .03	+ .01
2...	− .08	.00	1.08	1.00			− .07	− .02
3...	+ .01	.00	1.00	1.01	1.01	.98	+ .01	− .03
4...	.00	.00	.96	.96			+ .05	+ .02
5...	− .05	.00	.99	.94			+ .02	+ .04
6...	+ .07	.00	.94	1.01			+ .06	− .04
7...	+ .02	.00	.94	.96			+ .06	+ .01
8...	− .07	.00	1.02	.95	1.00	.97	− .02	+ .02
9...	− .11	.00	1.06	.95			− .06	+ .02
10...	− .09	.00	1.06	.97			− .06	.00

TABLE No. 23—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means.	
							Obs.	Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
July 11...	−0.05	0.00	1.03	0.98			−0.02	−0.02
12...	− .01	.00	1.00	.99			+ .01	− .03
13...	− .04	.00	.99	.95	1.01	0.96	+ .02	+ .01
14...	− .04	.00	1.00	.96			+ .01	.00
15...	− .08	.00	1.01	.93			.00	+ .03
16...	+ .03	.00	.90	.93			+ .03	− .04
17...	− .02	.00	.92	.90			+ .01	− .01
18...	− .08	.00	.94	.86	.93	.89	− .01	+ .03
19...	− .04	.00	.91	.87			+ .02	+ .02
20...	− .07	.00	.98	.91			− .05	− .02
21...	− .02	.00	.91	.89			+ .02	− .02
22...	− .05	.00	.90	.85			+ .03	+ .02
23...	− .07	.00	.97	.90	.93	.87	− .04	− .03
24...	− .12	.00	1.01	.89			− .08	− .02
25...	− .02	.00	.85	.83			+ .08	+ .04
26...	− .03	.00	.90	.87			+ .02	− .01
27...	− .07	.00	.92	.85			.00	+ .01
28...	− .10	.00	.94	.84	.92	.86	− .02	+ .02
29...	− .06	.00	.91	.85			+ .01	+ .01
30...	− .07	.00	.93	.86			− .01	.00
31...	− .06	.00	.93	.87			− .01	− .01
Aug. 1...	− .08	.00	.94	.86			− .05	.00
2...	− .06	.00	.93	.87			− .04	−.01
3...	+ .04	.00	.80	.84	.89	.86	+ .09	+ .02
4...	− .03	.00	.88	.85			+ .01	+ .01
5...	− .03	.00	.92	.89			− .03	− .03
6...	+ .01	.00	.88	.89			+ .01	.00
7...	− .04	.00	.94	.90			− .05	− .01
8...	− .03	.00	.92	.89	.89	.89	− .03	.00
9...	− .04	.00	.92	.88			− .03	+ .01
10...	+ .09	.00	.78	.87			+ .11	+ .02
11...	+ .06	.00	.82	.88			+ .05	.00
12...	+ .03	.00	.86	.89			+ .01	− .01
13...	− .03	.00	.90	.87	.87	.88	− .03	+ .01
14...	− .02	.00	.89	.87			− .02	+ .01
15...	+ .01	.00	.89	.90			− .02	− .02
16...	− .02	.00	.92	.90			− .05	− .02
17...	+ .01	.00	.89	.90			− .02	− .02
18...	+ .06	.00	.80	.86	.87	.88	+ .07	+ .02
19...	+ .03	.00	.85	.88			+ .02	.00
20...	− .02	.00	.90	.88			− .03	.00

TABLE No 23—*Continued.*

Date.	Barometric correction.	Wind correction.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means. Obs.	Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Aug. 21...	−0.11	0.00	0.98	0.87			−0.16	−0.05
22...	+ .05	.00	.73	(.78)			+ .09	(+ .04)
23...	+ .04	.00	.78	.82	0.82	0.82	+ .04	.00
24...	+ .02	.00	.77	.79			+ .05	+ .03
25...	− .03	.00	.82	.79			.00	+ .03
26...	− .04	.00	.80	.76			− .09	− .07
27...	− .06	.00	.80	.74			− .09	− .05
28...	+ .01	.00	.67	.68			+ .04	+ .01
29...	+ .05	.00	.63	.68	.71	.69	+ .08	+ .01
30...	.00	.00	.64	.64			+ .07	+ .05
31...	− .05	.00	.70	.65			+ .01	+ .04
Sept. 1...	− .05	.00	.71	.66			− .02	− .01
2...	− .02	.00	.64	.62			+ .05	+ .03
3...	+ .03	.00	.68	(.71)	.69	.65	+ .01	(− .06)
4...	.00	.00	.67	.67			+ .02	− .02
5...	.00	.00	.73	(.73)			− .04	(− .08)
6...	+ .08	.00	.53	.61			+ .04	− .01
7...	+ .08	.00	.49	.57			+ .08	+ .03
8...	+ .03	.00	.58	.61	.57	.60	− .01	− .01
9...	− .02	.00	.64	.62			− .07	− .02
10...	− .03	.00	.63	.60			− .06	.00
11...	− .05	.00	.65	.60			− .03	− .02
12...	+ .05	.00	.52	.57			+ .10	+ .01
13...	+ .01	.00	.57	.58	.62	.58	+ .05	.00
14...	− .09	.00	.69	.60			− .07	− .02
15...	− .12	.00	.68	.56			− .06	+ .02
16...	− .02	.00	.58	.56			+ .02	+ .02
17...	− .01	.00	.63	.62			− .03	− .04
18...	− .09	.00	.69	.60	.60	.58	− .09	− .02
19...	+ .01	.00	.56	.57			+ .04	+ .01
20...	+ .01	.00	.56	.57			+ .04	+ .01
21...	+ .03	.00	.55	.58			+ .02	− .02
22...	− .10	.00	.64	.54			− .07	+ .02
23...	− .11	.00	.71	.60	.57	.56	− .14	− .04
24...	+ .07	.00	.46	.53			+ .11	+ .03
25...	+ .06	.00	.51	.57			+ .06	− .01
26...	− .03	.00	.57	.54			− .07	− .03
27...	.00	.00	.52	.52			− .02	− .01
28...	.00	.00	.46	.46	.50	.51	+ .04	+ .05
29...	+ .04	.00	.48	.52			+ .02	− .01
30...	.00	.00	.49	.49			+ .01	+ .02

TABLE No. 24—*Observed and corrected elevations for the whole surface of Lake Erie as derived from Buffalo and Cleveland observations combined.*

Date.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means.	
					Obs.	Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
June 1.......	2.76	2.60			−0.15	0.00
2.......	2.59	2.63			+ .02	− .03
3.......	2.64	2.60	2.61	2.60	− .03	.00
4.......	2.48	2.58			+ .13	+ .02
5.......	2.56	2.59			+ .05	+ .01
6.......	2.64	2.56			− .04	+ .03
7.......	2.68	2.56			− .08	+ .03
8.......	2.64	2.59	2.60	2.59	− .04	.00
9.......	2.54	2.62			+ .06	− .03
10.......	2.49	2.60			+ .11	− .01
11.......	2.52	2.60			+ .10	− .01
12.......	2.68	2.56			− .06	+ .03
13.......	2.63	2.59	2.62	2.59	− .01	.00
14.......	2.62	2.58			.00	+ .01
15.......	2.63	2.60			− .01	− .01
16.......	2.59	2.59			+ .02	− .01
17.......	2.62	2.56			− .01	+ .02
18.......	2.64	2.57	2.61	2.58	− .03	+ .01
19.......	2.61	2.60			.00	− .02
20.......	2.58	2.58			+ .03	.00
21.......	2.58	2.59			− .12	− .03
22.......	2.56	2.56			− .10	.00
23.......	2.57	2.56	2.46	2.56	− .11	.00
24.......	2.20	2.47			+ .26	+ .09
25.......	2.39	2.60			+ .07	− .04
26.......	2.41	2.40			+ .10	+ .03
27.......	2.59	2.36			− .08	+ .07
28.......	2.60	2.50	2.51	2.43	− .09	− .07
29.......	2.48	2.43			+ .03	.00
30.......	2.48	2.45			+ .03	− .02
July 1.......	2.48	2.40			− .09	− .02
2.......	2.44	2.41			− .05	− .03
3.......	2.46	2.39	2.39	2.38	− .07	− .01
4.......	2.29	2.36			+ .10	+ .02
5.......	2.26	2.36			+ .13	+ .02
6.......	2.33	2.38			+ .05	− .02
7.......	2.40	2.36			− .02	.00
8.......	2.38	2.38	2.38	2.36	.00	− .02
9.......	2.38	2.35			.00	+ .01
10.......	2.39	2.33			− .01	+ .03

TABLE No. 24—*Continued.*

Date.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
July 11	2.42	2.32			+0.01	+0.08
12	2.36	2.37			+ .07	+ .03
13	2.50	2.41	2.43	2.40	− .07	− .01
14	2.44	2.46			− .01	− .06
15	2.45	2.45			− .02	− .05
16	2.38	2.17			− .06	+ .06
17	2.30	2.09			+ .02	+ .14
18	2.28	2.35	2.32	2.23	+ .04	− .12
19	2.33	2.34			− .01	− .11
20	2.29	2.20			+ .03	+ .03
21	2.34	2.29			+ .05	− .04
22	2.38	2.31			+ .01	− .06
23	2.30	2.32	2.39	2.25	+ .09	− .07
24	2.42	2.20			− .03	+ .05
25	2.52	2.13			− .13	+ .12
26	2.44	2.39			− .03	− .02
27	2.41	2.38			.00	− .01
28	2.38	2.36	2.41	2.37	+ .03	+ .01
29	2.38	2.37			+ .03	.00
30	2.46	2.40			− .05	− .03
31	2.38	2.31			+ .03	+ .06
Aug. 1	2.36	2.36			+ .06	− .01
2	2.34	2.37			+ .08	− .02
3	2.30	2.34	2.42	2.35	+ .12	+ .01
4	2.52	2.37			−. 10	− .02
5	2.56	2.31			− .14	+ .04
6	2.36	2.29			− .11	− .03
7	2.20	2.25			+ .05	+ .01
8	2.20	2.24	2.25	2.26	+ .05	+ .02
9	2.18	2.24			+ .07	+ .02
10	2.29	2.26			− .04	.00
11	2.26	2.26			− .11	− .10
12	2.15	2.21			.00	− .05
13	2.08	2.04	2.15	2.16	+ .07	+ .12
14	2.10	2.07			+ .05	+ .09
15	2.14	2.20			+ .01	− .04
16	2.12	2.21			+ .02	.00
17	2.11	2.21			+ .03	.00
18	2.20	2.22	2.14	2.21	− .06	− .01
19	2.17	2.20			− .03	+ .01
20	2.12	2.19			+ .02	+ .02

TABLE No. 24—*Continued.*

Date.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means.	
					Obs.	Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Aug. 21.......	2.07	2.16	2.11	2.12	+0.04	−0.04
22.......	2.10	2.15			+ .01	− .03
23.......	2.06	2.10			+ .05	+ .02
24.......	2.06	2.10			+ .05	+ .02
25.......	2.24	2.11			− .13	+ .01
26.......	2.21	2.15	1.98	2.08	− .23	− .07
27.......	2.04	2.03			− .06	+ .05
28.......	1.96	2.03			+ .02	+ .05
29.......	1.77	2.18			+ .21	− .10
30.......	1.90	2.05			+ .08	+ .03
31.......	2.00	2.02			− .02	+ .06
Sept. 1.......	1.98	2.01	2.06	2.11	+ .08	+ .10
2.......	2.02	2.11			+ .04	.00
3.......	2.05	2.14			+ .01	− .03
4.......	2.20	2.19			− .14	− .08
5.......	2.04	2.10			+ .02	+ .01
6.......	2.23	2.13	2.10	2.06	− .13	− .07
7.......	2.10	2.03			.00	+ .03
8.......	2.06	2.09			+ .04	− .03
9.......	2.15	2.06			− .05	.00
10.......	1.95	1.99			+ .15	+ .07
11.......	1.96	2.01	1.93	1.94	− .03	− .07
12.......	1.97	1.98			− .04	− .04
13.......	1.92	1.91			+ .01	+ .03
14.......	1.90	1.89			+ .03	+ .05
15.......	1.90	1.89			+ .03	+ .05
16.......	1.71	1.76	1.81	1.85	+ .10	+ .09
17.......	1.80	1.73			+ .01	+ .12
18.......	1.88	1.88			− .07	− .03
19.......	1.75	2.01			+ .06	− .16
20.......	1.90	1.89			− .09	− .04
21.......	1.86	1.87	1.79	1.88	− .07	+ .01
22.......	1.66	1.98			+ .13	− .10
23.......	1.78	1.87			+ .01	+ .01
24.......	1.71	1.85			+ .08	+ .03
25.......	1.92	1.85			− .13	+ .03
26.......	1.52	1.62	1.78	1.79	+ .26	+ .17
27.......	1.98	1.92			− .20	− .13
28.......	1.86	1.82			− .08	− .03
29.......	1.74	1.78			+ .04	+ .01
30.......	1.80	1.79			− .02	.00

Table No. 24—*Continued.*

Date.	Observed elevation 570+.	Corrected elevation 570+.	5-day observed mean 570+.	5-day corrected mean 570+.	Residuals from 5-day means.	
					Obs.	Cor.
1910.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Oct. 1	2.10	1.87			−0.29	−0.04
2	1.70	1.77			+ .11	+ .06
3	1.68	1.78	1.81	1.83	+ .13	+ .05
4	1.79	1.76			+ .02	+ .07
5	1.76	1.96			+ .05	− .13
6	2.07	1.73			− .09	+ .15
7	1.90	1.92			+ .08	− .04
8	1.93	1.93	1.98	1.88	+ .05	− .05
9	2.08	1.94			− .10	− .06
10	1.94	1.88			+ .04	.00
11	2.05	1.83			− .21	− .01
12	1.80	1.86			+ .04	− .04
13	1.50	1.71	1.84	1.82	+ .34	+ .11
14	1.90	1.84			− .06	− .02
15	1.94	1.85			− .10	− .03
16	1.93	1.80			− .10	.00
17	1.81	1.85			+ .02	− .05
18	1.85	1.86	1.83	1.80	− .02	− .06
19	1.78	1.69			+ .05	+ .11
20	1.76	1.82			+ .07	− .02
21	1.72	2.01			+ .23	− .15
22	2.34	....			− .39	....
23	1.98	1.87	1.95	1.86	− .03	− .01
24	1.84	1.76			+ .11	+ .10
25	1.89	1.81			+ .06	+ .05
26	1.77	1.78			+ .10	+ .04
27	2.18	1.80			− .31	+ .02
28	1.85	1.86			+ .02	− .04
29	1.88	1.85	1.87	1.82	− .01	− .03
30	1.84	....			+ .03	....
31	1.72	1.81			+ .15	+ .01

In table No. 24 the corrected elevation for any day is the weighted mean of the corrected elevation for Buffalo as recorded in table No. 19 and the corrected elevation for Cleveland as recorded in table No. 20, the relative weights assigned being 1.0 for Buffalo and 1.7 for Cleveland.

The general explanations given on pages 78–79 with reference to 5-day means and residuals in table No. 19 also apply to table No. 24.

MEAN ELEVATIONS OF LAKE MICHIGAN–HURON.

The general explanations made in connection with table No. 24 for Lake Erie apply to table No. 25 for Lake Michigan–Huron.

TABLE No. 25—*Observed and corrected elevations for the whole surface of Lake Michigan-Huron as derived from Milwaukee, Harbor Beach, and Mackinaw observations combined.*

Date.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means. Obs.	Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
June 1.......	0.92	0.93			+0.01	−0.01
2.......	.91	.92			+ .02	.00
3.......	.92	.90	0.93	0.92	+ .01	+ .02
4.......	.95	.92			− .02	.00
5.......	.96	.93			− .03	− .01
6.......	.97	.94			+ .01	+ .01
7.......	.96	.94			+ .02	+ .01
8.......	.99	.96	.98	.95	− .01	− .01
9.......	.98	.95			.00	.00
10.......	.98	.97			.00	− .02
11.......	1.00	1.01			+ .02	+ .01
12.......	1.02	1.04			.00	− .02
13.......	1.03	1.02	1.02	1.02	− .01	.00
14.......	1.03	1.03			− .01	− .01
15.......	1.03	1.02			− .01	.00
16.......	1.04	1.04			.00	.00
17.......	1.06	1.06			− .02	− .02
18.......	1.04	1.03	1.04	1.04	.00	+ .01
19.......	1.03	1.04			+ .01	.00
20.......	1.05	1.04			− .01	.00
21.......	1.05	1.04			− .02	+ .01
22.......	1.04	1.05			− .01	.00
23.......	1.01	1.06	1.03	1.05	+ .02	− .01
24.......	1.01	1.10			+ .02	− .05
25.......	1.03	1.01			.00	+ .04
26.......	1.05	1.02			− .03	− .02
27.......	1.00	1.00			+ .02	.00
28.......	1.02	1.01	1.02	1.00	.00	− .01
29.......	1.02	.99			.00	+ .01
30.......	1.01	.99			+ .01	+ .01
July 1.......	1.00	1.00			.00	.00
2.......	1.00	1.01			.00	− .01
3.......	1.01	1.01	1.00	1.00	− .01	− .01
4.......	1.00	.99			.00	+ .01
5.......	.99	.98			+ .01	+ .02
6.......	1.02	1.02			− .02	− .02
7.......	1.03	1.00			− .03	.00
8.......	1.01	1.00	1.00	1.00	− .01	.00
9.......	.97	.99			+ .03	+ .01
10.......	.98	.97			+ .02	+ .03

TABLE No. 25—*Continued.*

Date.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
July 11	0.96	1.00			−0.01	−0.02
12	.96	.99			− .01	− .01
13	.95	.97	0.95	0.98	.00	+ .01
14	.96	.98			− .01	.00
15	.94	.95			+ .01	+ .03
16	.94	.94			− .02	− .02
17	.94	.94			− .02	− .02
18	.91	.91	.92	.92	+ .01	+ .01
19	.89	.90			+ .03	+ .02
20	.91	.93			+ .01	− .01
21	.87	.90			− .02	− .02
22	.88	.89			− .03	− .01
23	.88	.90	.85	.88	− .03	− .02
24	.78	.88			+ .07	.00
25	.85	.82			.00	+ .06
26	.87	.88			− .01	− .02
27	.86	.86			.00	.00
28	.85	.86	.86	.86	+ .01	.00
29	.85	.86			+ .01	.00
30	.85	.86			+ .01	.00
31	.86	.87			.00	− .01
Aug. 1	.85	.86			+ .01	.00
2	.87	.87			− .01	− .01
3	.85	.84	.86	.86	+ .01	+ .02
4	.86	.85			.00	+ .01
5	.86	.87			.00	− .01
6	.87	.87			.00	.00
7	.87	.87			.00	.00
8	.85	.86	.87	.87	+ .02	+ .01
9	.86	.87			+ .01	.00
10	.90	.88			− .03	− .01
11	.92	.89			− .03	.00
12	.89	.88			.00	+ .01
13	.89	.88	.89	.89	.00	+ .01
14	.89	.89			.00	.00
15	.88	.89			+ .01	.00
16	.90	.91			.00	− .01
17	.91	.92			− .01	− .02
18	.90	.90	.90	.90	.00	.00
19	.91	.90			− .01	.00
20	.89	.89			+ .01	+ .01

TABLE No. 25—*Continued.*

Date.	Observed elevation 579+.	Corrected elevation 579+.	5-day observed mean 579+.	5-day corrected mean 579+.	Residuals from 5-day means. Obs.	Residuals from 5-day means. Cor.
1911.	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*	*feet.*
Aug. 21	0.85	0.87			−0.02	−0.03
22	.84	.89			− .01	− .05
23	.84	.84	0.83	0.84	− .01	.00
24	.81	.80			+ .02	+ .04
25	.81	.81			+ .02	+ .03
26	.79	.79			− .05	− .07
27	.76	.77			− .02	− .05
28	.73	.73	.74	.72	+ .01	− .01
29	.77	.70			− .03	+ .02
30	.69	.68			+ .05	+ .04
31	.69	.68			+ .05	+ .04
Sept. 1	.68	.68			.00	− .01
2	.67	.68			+ .01	− .01
3	.72	.70	.68	.67	− .04	− .03
4	.68	.66			.00	+ .01
5	.67	.64			+ .01	+ .03
6	.70	.66			− .02	.00
7	.70	.64			− .02	+ .02
8	.67	.68	.68	.66	+ .01	− .02
9	.67	.66			+ .01	.00
10	.66	.65			+ .02	+ .01
11	.64	.64			.00	− .01
12	.70	.63			− .06	.00
13	.66	.64	.64	.63	− .02	− .01
14	.61	.62			+ .03	+ .01
15	.59	.60			+ .05	+ .03
16	.61	.61			+ .02	+ .01
17	.66	.64			− .03	− .02
18	.64	.63	.63	.62	− .01	− .01
19	.62	.61			+ .01	+ .01
20	.63	.61			.00	+ .01
21	.61	.60			.00	+ .01
22	.62	.60			− .01	+ .01
23	.61	.62	.61	.61	.00	− .01
24	.62	.59			− .01	+ .02
25	.60	.62			+ .01	− .01
26	.60	.57			− .01	− .02
27	.57	.56			+ .02	− .01
28	.59	.55	.59	.55	.00	.00
29	.60	.52			− .01	+ .03
30	.58	.55			+ .01	.00

In table No. 25 the observed elevation for any day was obtained by merely taking a mean of the three observed elevations for that day as recorded in table No. 21 for Milwaukee, in table No. 22 for Harbor Beach, and in table No. 23 for Mackinaw.

In table No. 25 the corrected elevation for any day is the weighted mean of the three corrected elevations as given in tables No. 21, 22, and 23, the relative weights assigned being 1.9 for Milwaukee, 2.9 for Harbor Beach, and 5.1 for Mackinaw.

EXPLANATION OF PLATES 7 to 13.

Plates 7 to 13, inclusive, show graphically a considerable part of the information which is given in numerical form in tables Nos. 19 to 25, inclusive. These graphs are especially valuable as a means of showing the increase in accuracy which has been secured by applying corrections for wind effects and barometric effects. The desired values are the elevations of the mean surface of the whole of a lake from day to day. Consider the evidence in these graphs that the corrected elevations are much closer approximations to the desired elevations of the mean surface of the whole lake from day to day than are the directly observed elevations.

Plates 7, 8, 9, and 10 combined show continuous graphs for Lake Erie from June 1, 1910, to October 31, 1910. The graphs on the lower half of each of the plates 7, 8, and 9 are continuations of those on the upper half of the same plate, and are continued, in turn, by those on the upper half of the next plate. Plate 10 is a continuation from plate 9 on the same scale. On account of the extreme fluctuations in observed elevations in the last 10 days of October, it was necessary to use the whole of plate 10 for a single set of graphs.

Similarly, plates 11 to 13 combined show continuous graphs for Lake Michigan–Huron from June 1, 1911, to September 30, 1911.

Throughout these plates 7 to 13 the elevations directly observed at each gage, or means of such elevations from the two or three gages on each lake, are indicated by a dash-and-dot line—a long dash and a dot alternately.

The corrected elevations from each gage, from the weighted mean of the two or three gages on a lake, are indicated by a continuous solid line.

The difference between the dash-and-dot line and the continuous line has been produced by the application of corrections for wind effects and barometric effects. The question on which it is desired to concentrate attention at present is, How much more accurately in each case does the continuous line represent the fluctuation in elevation of the mean lake surface than the dash-and-dot line?

The figures showing elevations, at the right-hand margin of each plate, are feet and refer to mean sea-level as a datum.

The computed fluctuation in elevation of the mean lake surface, in each case, as produced by the rainfall on the lake surface, the inflow into the lake from the next lake above in the chain of Great Lakes, and the outflow to the

next lake below in the chain, is shown for each lake by a line made up of successive groups of two short dashes and a dot. This line is plotted from the measured values of the rainfall on the lake surface, as determined by rain gages around the lake, and from the measured inflow and outflow in the streams connecting the chain of Great Lakes. Note the smoothness of this line. In general, it rises or falls less than 0.02 foot in a day, usually not more than 0.01 foot. In other words, the fluctuation of the elevation of the mean surface of the lake due to rainfall, inflow, and outflow is normally at the very slow rates stated. The extreme case anywhere on these graphs is the rise of 0.19 foot in 3 days, October 4–7, 1910, of Lake Erie, due to extremely heavy rainfall over the whole lake surface on those days.

ACCURACY AS TESTED BY GRAPHS.

The mean surface of the whole of any one of the Great Lakes changes only as the total content of the lake changes. That total content changes from five causes only: (1) rainfall on the lake surface, (2) inflow from the next lake above, (3) outflow to the next lake below, (4) run-off into the lake from the surrounding land-drainage area, and (5) evaporation from the lake surface.

The dot-and-two-dashes graph, commented upon above and based upon direct measurement, shows how slow and regular is the variation of elevation of the mean lake surface due to the first three of the causes enumerated in the preceding paragraph. It is believed that the variation in elevation due to the fourth and fifth causes—run-off and evaporation—is even smaller and more regular than that due to the first three. The run-off and the evaporation in question have not been measured directly. From sources of information which are in part external to this investigation, it is estimated that during the months June to October of each year the run-off into Lake Erie from the surrounding land-drainage area is such as to produce a rise from 0.004 to 0.040 foot per day in the mean lake surface, with only a small percentage of days in which the rise is more than 0.020 foot. For Lake Michigan–Huron the run-off expressed in the same terms is even more constant. So, too, from external evidence, it is estimated that on either lake during the season June to October the evaporation produces a fall in the mean lake surface varying from but little more than 0.000 foot on some days to 0.021 foot on days of extremely rapid evaporation. The considerations indicated in this paragraph lead to the belief that the actual variation in the elevation in the mean surface of either Lake Erie or Lake Michigan–Huron is so slow and regular as to be properly represented by a graph but slightly less smooth and regular than the dot-and-two-dashes graph shown in plates 7 to 13. It is believed that the actual variation of the mean elevation of the whole surface of either lake is as a rule about 0.01 foot or less in a day, is sometimes somewhat more that 0.02 foot in a day, and only on very rare occasions exceeds 0.08 foot in any one day.

Hence, apparent fluctuations in the graphs of observed elevations or of corrected elevations which exceed the rates stated in the last sentence of the preceding paragraph are evidence that said graphs are imperfect representations of the rise and fall of the mean surface of the lake. The nearer a given graph approaches to the smoothness of the dot-and-two-dashes graph, or to the rates of fluctuations stated in the last sentence of the preceding paragraph, the more accurately does it represent the fluctuation of the mean surface of the lake. If one examines the graphs of observed elevations and of corrected elevations on plates 7 to 13 with this point of view, it becomes clearly evident that the corrected elevations are much more accurate than the observed elevations.

Examine first the Buffalo graphs on plates 7 to 10.

Even for June 1 to July 15, 1910, a comparatively quiet part of the year, when the wind effects and barometric effects at Buffalo are much smaller than in October, note how much smoother is the continuous (corrected elevation) graph than the dot-and-one-dash (observed elevation) graph on plate 7. Note, for example, the following contrast: On June 1–4 the observed elevation decreased 0.67 foot in three days, immediately increased 0.50 foot in the three days June 4–7, then decreased 0.51 foot in the three days June 7–10, and then increased 0.52 foot in the two days June 10–12. The changes in the corrected elevations in these same successive periods were respectively a decrease of 0.12 foot, an increase of 0.11 foot, no change, and a decrease of 0.15 foot. The fluctuations in the corrected graph were not more than one-fourth as rapid as on the observed graph in the interval June 1–12. During the period June 1–July 15 the greatest change in the observed elevation (dot-and-dash graph) in any single day was 0.50 foot decrease on July 3–4. During the same period as covered by plate 7 the greatest change in a single day in the corrected elevation, as shown by the continuous graph, was a decrease of 0.20 foot June 25–26. It is evident that the corrected elevations at Buffalo do not represent the fluctuations in the mean surface of Lake Erie without error, for there are days on which the corrected elevation changes more rapidly than it is possible for the elevation of the mean surface of Lake Erie to change. But it is equally evident from the graphs that the corrected elevations are a much more accurate representation of the fluctuations of the mean surface than are the observed elevations. From an inspection of plate 7, one may conclude that the errors in the corrected elevations are from one-half to one-quarter as large as those in the observed elevations.

Consider next the Buffalo graphs on plate 8. In general, one finds the same kind of contrasts as on plate 7. Note, for example, the range of 0.69 foot in the observed elevations in the period July 17–25, 1910. In the same period the total range of the corrected elevations was only 0.21 foot. Note, also, the extremely high points on the dot-and-dash graph on August 5 and August 25 and the relative smoothness of the continuous graph near those points.

On plate 9, examine the Buffalo graphs for the period September 18–October 4. Note the extreme irregularity of the dot-and-dash graph (observed elevations), showing an extreme change of 0.87 foot on October 1–2, and the relative smoothness of the continuous graph (corrected elevations), with no change in a single day of more than 0.12 foot.

Plate 10 covers a stormy period, October 10–31, 1910. Note the extreme irregularity of the observed elevations at Buffalo. On October 21–22 the observed elevation increased 1.75 feet in a single day. The corrected elevation shows no change greater than 0.10 feet in any single day.

Consider the Cleveland graphs. On plates 7 and 8, though the continuous graph is smoother, as a whole, than the dot-and-dash graph, the contrast is not great. On plate 9 the small fluctuations in the continuous graph for the periods August 28 to September 10 and September 29 to October 9, 1910, are accompanied by much larger fluctuations in the dot-and-dash graph. On plate 10 the same contrast, indicating much greater accuracy for the corrected elevations than for the observed elevations, shows for the period October 21–31, 1910.

As the corrections for wind effects and barometric effects at Cleveland are in general much smaller than at Buffalo, it is to be expected that the improvement in accuracy produced by applying the corrections will be less noticeable at Cleveland than at Buffalo, and hence that the contrast between the dot-and-dash graph and the continuous graph will be much less pronounced at Cleveland that at Buffalo.

So, too, one may naturally expect the contrast between the two graphs to be still less for the Lake Erie means as shown on the third pair of graphs on plates 7 to 10 than for either Buffalo or Cleveland. Nevertheless, the contrast even on these curves shows clearly in favor of the corrected elevations. In this connection, note especially the decided smoothness of the continuous graph for the Lake Erie mean for June 1–23, 1910, on plate 7 as compared with the dot-and-dash graph, and the same contrast on plate 10 for the stormy period October 11–31, 1910.

Compare the continuous graphs on plates 11, 12, and 13 with the dot-and-dash graphs. For each of the three separate gages—Mackinaw, Harbor Beach, and Milwaukee—it is clear that the continuous graph is decidedly smoother, on the whole, than the dot-and-dash graph. In this connection, attention is especially invited to the period June 23–July 15, 1911, shown on the lower half of plate 11; to the period July 22–26, 1911, shown on plate 12; and to the period August 28–September 19, 1911, shown on plate 13. The dot-and-dash graph for Lake Michigan–Huron mean is a remarkably smooth curve. It so closely approaches in smoothness to the dot-and-two-dashes curve for the same lake (the rainfall + inflow − outflow graph) that it is necessarily difficult to tell whether the continuous graph is intermediate in smoothness. The graphic method of comparison is not sufficiently sensitive to determine reliably the relative accuracy in this case, in which both graphs, for the Lake Michigan–Huron

mean observed elevations and for the Lake Michigan–Huron mean corrected elevations, are of a high degree of accuracy. The determination of relative accuracy must be made mainly by other methods in this case. However, an examination of the two graphs makes it clear that both are of a very high degree of accuracy.

MONTHLY AND SEASONAL MEAN ELEVATIONS.

The following tables, No. 26 for Lake Erie and No. 27 for Lake Michigan–Huron, show the monthly and seasonal mean observed elevations and mean corrected elevations of the water surface, first, for the separate gage stations, and in the last two columns for the lake as derived from the means from the stations named. These means—monthly and seasonal—are based directly on the values shown in tables Nos. 19 to 25, and on similar tables, which are not published, for the season of 1909 on Lake Erie and the season of 1910 on Lake Michigan–Huron.

TABLE No. 26.

[Mean observed and mean corrected elevation = 570+ feet.]

Year and month.	Buffalo gage.		Cleveland gage.		Lake Erie.	
	Observed elevation.	Corrected elevation.	Observed elevation.	Corrected elevation.	Observed elevation.	Corrected elevation.
1909, Aug......	2.68	2.63	2.80	2.80	2.75	2.75
1909, Sept......	2.30	2.25	2.36	2.36	2.31	2.32
1909, Oct......	2.05	1.82	1.76	1.82	1.90	1.81
1910, June.....	2.55	2.49	2.61	2.59	2.57	2.56
1910, July.....	2.37	2.26	2.40	2.42	2.39	2.33
1910, Aug......	2.13	2.06	2.22	2.28	2.18	2.20
1910, Sept......	1.83	1.80	2.02	2.01	1.91	1.94
1910, Oct......	1.92	1.70	1.88	1.92	1.88	1.84
1909, whole season, Aug.–Oct.........	2.34	2.23	2.31	2.33	2.32	2.29
1910, whole season, June–Oct.........	2.16	2.06	2.22	2.24	2.19	2.17

PROBABLE ERRORS AND WEIGHTS.

The residuals as tabulated in tables Nos. 19 to 25, inclusive, are evidently a test of the accuracy of the observed elevations and of the corrected elevations corresponding to these residuals. It is desirable to study these residuals and formulate the conclusions from them.

If N independent determinations are made of a quantity which is constant, and the residuals are taken of the separate determinations from the mean of

the N determinations, then the probable error of one determination may be computed from the formula

$$\frac{2}{3}\sqrt{\frac{\Sigma v^2}{N-1}}$$

in which Σv^2 stands for the sum of the squares of the residuals in the group.

TABLE No. 27.

[Mean observed and mean corrected elevation = 579+ feet.]

Year and month.	Milwaukee gage.		Harbor Beach gage		Mackinaw gage.		Lake Michigan-Huron.	
	Observed elevation.	Corrected elevation.	Observed elevation.	Corrected elevation.	Observed elevation.	Corrected elevation.	Observed elevation.	Corrected elevation.
1910, June .	1.56	1.59	1.54	1.50	1.48	1.49	1.52	1.50
1910, July .	1.49	1.51	1.42	1.41	1.41	1.41	1.44	1.43
1910, Aug. .	1.31	1.29	1.33	1.33	1.30	1.28	1.31	1.30
1910, Sept..	1.29	1.28	1.20	1.24	1.24	1.22	1.24	1.24
1911, June .	1.05	1.06	.98	.98	.98	.98	1.00	1.00
1911, July..	.89	.98	.93	.95	.97	.92	.93	.94
1911, Aug. .	.85	.87	.85	.85	.84	.84	.85	.85
1911, Sept..	.75	.74	.58	.61	.59	.58	.64	.62
1910, whole season...	1.41	1.42	1.37	1.37	1.36	1.35	1.38	1.37
1911, whole season...	0.89	.91	.84	.85	.84	.83	.86	.85

The probable error of a single observed elevation at each of the five stations was computed in turn from this formula applied to each group of N residuals as shown in the tables Nos. 19 to 23. It will be noted that N was usually 5, but was sometimes 6, for the group at the end of a month, and sometimes less than 5 where observations were missing or were rejected. The mean of these many values of the probable error of a single determination, one for each group, was taken as the probable error of a single determination for that station, of the kind under consideration, observed or corrected.

The probable errors so computed are shown in table No. 28.

When one considers the probable errors for an observed elevation as shown for each of the five gage stations in table No. 28, the question naturally arises, Why should the observations at the different stations on any lake be given equal weight? Two considerations led to the decision to assign equal weights to the observed elevations in this investigation. First, that has been the usual practice, so far as the investigator knows, in connection with other studies on the Great Lakes. It did not seem desirable to depart from past practice except for clearly good reasons. Second, it was clearly

evident, throughout this investigation, that the observations at any one gage, considered as an attempt to secure the mean elevation of the whole lake surface, are subject to systematic errors which are not small in comparison with the accidental errors, and that, therefore, the computed probable error of a single observed elevation, such a probable error as those shown in table No. 28, is unreliable as an indication of relative accuracy at different stations. Among the systematic errors to which the observed elevations are subject are the barometric effects and wind effects. It is not at all certain that these and other systematic errors would be more thoroughly eliminated from the mean observed elevations by using weights based on the probable errors than by using equal weights at the different stations. In fact, which procedure would give the best elimination depends largely upon the relations to each other of the systematic errors at the various stations as well as upon the accidental errors. Incidentally, it is interesting to note here that evidence developed very late in this investigation as to the barometric effects on Lake Michigan–Huron shows clearly that the algebraic sum of the barometric effects at the three stations—Milwaukee, Harbor Beach, and Mackinaw—tends to be nearly zero on each day.* Therefore, it is clear that equal weights assigned to the three observed elevations on any day at these stations tends to give a much better elimination of the principal systematic error—that from barometric effects—than if unequal weights were used.

Table No. 28.—*Probable error of a single daily elevation of the water surface computed from the residuals shown in tables 19 to 23.*

	For an observed elevation.	For a corrected elevation.
	feet.	*feet.*
Buffalo gage	±.147	±.036
Cleveland gage	±.059	±.028
Milwaukee gage	±.054	±.026
Harbor Beach gage	±.056	±.021
Mackinaw gage	±.039	±.016

On the other hand, when one tests the corrected elevations in various ways, little evidence is found of systematic errors. The errors in these values seem to be mainly of an accidental character. Hence, for corrected elevations each station has been given a weight inversely proportional to the square of its probable error as shown in table No. 28. Unit weight was assigned to Buffalo, where the probable error is ±0.036 foot. The weights assigned on this basis for other stations are stated on pages 100, 104.

* Consult table No. 7, page 33, and note that the algebraic sum of the three values of C_w for Milwaukee, Harbor Beach, and Mackinaw, namely, -4.97, $+6.94$, and -2.54, is only -0.57. Similarly, note that the algebraic sum of the three values of C_n is only $+3.62$. The smaller these two sums the smaller the sum of the barometric effects at the stations tends to be on each day.

Among the values of the corrected elevations shown in tables Nos. 19 to 23 there is an occasional one (inclosed in parentheses) which has been rejected by a definite criterion, as referred to on page 79, and therefore has no influence upon the five-day mean or any of the other later means, monthly or seasonal. That criterion is of the character already described in general terms on page 73 in connection with wind effects and on page 66 in connection with barometric effects. It is intended to identify values which are decidedly abnormal, due to an influence which extends over a single day only, believed usually to be due either to the first (and very large) half wave of a new seiche, started by a new powerful wind impulse or barometric impulse, or to a wide departure of fact from the assumptions used in computing the barometric effects. Such wide departures are believed to occur, as a rule, when a well-developed area of low pressure is over or near the lake.

The criterion as applied to the corrected elevations shown in tables Nos. 19 to 23 was as follows:

A corrected elevation for a given day is to be rejected whenever it differs from that for the next preceding or next following day by more than 3.5 times the probable error of the change for one day as computed in the final least-square solution for barometric effects at that station, provided, also, that after such rejection the apparent change of elevation then showing for the two-day interval covering the rejection is inside the 3.5 limit just stated.

The limit used in the criterion for each station—3.5 times the probable error named—was, in feet, for Buffalo ± 0.15, for Cleveland ± 0.14, for Milwaukee ± 0.10, for Harbor Beach ± 0.09, and for Mackinaw ± 0.08.

In a few rare cases the criterion gave somewhat ambiguous results. In these cases the ambiguity was removed by considering the residuals from the five-day means.

This criterion is believed to be reasonably reliable as a means of identifying abnormal values, and thus improving the final accuracy by preventing any influence being carried forward from these abnormal values into the final computed values.

Attention was called on page 108 to the fact that the method of computing the probable errors there indicated is based upon the assumption that the quantity observed is a constant. Consider this assumption in connection with the corrected elevations. The corrected elevation is supposed to be the elevation of the mean surface of the lake in question. Said elevation of the mean surface of the lake is certainly not a constant. It varies as the total water content of the lake changes. It is therefore subject to continual changes due to rainfall on the lake surface, inflow into the lake from the next lake above in the chain of Great Lakes, outflow from the lake to the next lake below in the chain, run-off into the lake from the surrounding land, and evaporation from the lake surface. Hence, the elevation of the mean lake surface is in general continually fluctuating, due to the causes enumerated. The residuals from the five-day means are thereby affected in such wise as to be increased on an average over what they would otherwise

be. The computed probable errors are therefore somewhat too large to represent the true degree of accuracy of a single corrected elevation.

From various sources of information, some of them external to this investigation, the following estimates of the fluctuations of mean lake surface of Lake Erie have been made. The estimates are approximately applicable to Lake Michigan–Huron, with the one exception that the net outflow from Lake Michigan–Huron is smaller, in the units here used, than from Lake Erie and less variable.

It is estimated for Lake Erie that during the months June to October of each year the net outflow, namely, the outflow through the Niagara River minus the inflow through the Detroit River, corresponds to a fall in the mean lake surface varying from +0.016 foot on some days to −0.002 foot at the other extreme for some days. Similarly, it is estimated that the rainfall on the lake surface causes a rise in the mean lake surface varying from zero on days of no rain to +0.085 on rare days of very heavy rainfall; that the run-off into the lake from the surrounding land corresponds to a rise varying from +0.004 foot on some days to +0.040 on other days; and that the evaporation corresponds to a fall in the mean lake surface of from 0.000 foot on some days to +0.021 foot on days of extremely rapid evaporation.

For the season of 1910, the five months June to October, inclusive, the algebraic sum of these four influences was a fall of about 0.004 foot per day for the whole season.

An inspection of the monthly means of corrected elevations as shown in table No. 26 makes it clear that in general during the season June to October the mean lake surface is falling at a mean rate usually between 0.00 and 0.02 foot per day.

It is clear from the three paragraphs next preceding this that the actual variation of the mean elevation of the whole surface of any one of the Great Lakes is, as a rule, as much as 0.01 foot in two days, that it is frequently more than 0.02 foot in 24 hours, and that on rare occasions it may exceed 0.08 in that period.

This estimated rate of variation was taken into account in fixing the size of the group to be used in taking each mean in such tables as Nos. 19 to 24. If the group were taken very small, as, for example, two or three days only, the mean would be unstable, and the two or three residuals would give a very poor computed value for the probable error. On the other hand, if the group were made large, to include, say, eleven values, then the extreme residuals in each group would depend very largely upon the actual variations in the mean lake surface commented upon in the four preceding paragraphs, and so would not be of value as indicators of the accuracy of the corrected elevation for each day. In an eleven-day group, the extreme residual at each end of the group would include the variation in elevation of the mean lake surface for five days, the interval between that residual and the middle date of the group. For the mean for the group most nearly represents the middle date in

general. The final decision was to use groups of five days each as being the best available compromise between the two difficulties. The end residuals of each five-day group include, in general, the actual fluctuation in mean lake elevation for two days only.

With the five-day groups, and on the basis of the estimate stated in the second paragraph before this, it appears that of the end residuals of each group 0.01 foot is usually due to actual fluctuation in the elevation of the mean surface of the lake. In a few cases as much as 0.04 foot or more of such end residuals may be due to such fluctuation.

From the considerations which have been indicated briefly above, the writer estimates that the probable errors shown in the last column of table No. 28 are appreciably too large to represent the true degree of accuracy of the corrected elevations. It is possible that the probable error of a corrected elevation for one day for the mean surface of the whole of Lake Michigan–Huron, as derived from observations at the Mackinaw gage, is as small as ± 0.010 foot, instead of the value ± 0.016 foot, as shown in table No. 28. Much better determinations of these probable errors will become available when the investigation of evaporation is made from this data. Until that time, one must be content with the approximate estimate of this paragraph and the positive knowledge that the values in the last column of table No. 28 are too large to represent the truth.

TIDES.

The question may properly be raised, Do the true tides produced in the Great Lakes by the moon and sun have any appreciable effect upon this investigation? No account has been taken of such tides anywhere in this investigation. The true tide at Chicago and at Milwaukee produces a total range of oscillation of 0.14 foot or less. (See Coast and Geodetic Survey Report for 1907, pages 483–486, in the Manual of Tides, by R. A. Harris.) This tidal oscillation is made up of several components, each approximately a sine wave, and each with a period which is either approximately 12 solar hours or 12 lunar hours, or 24 solar hours or 24 lunar hours. Such an oscillation has practically no effect on the mean for each day. Certainly it does not affect it by as much as 0.01 foot in any case. At other stations on the Great Lakes the tide is, as a rule, smaller than at Chicago and Milwaukee, and is nowhere much larger. The true tides produce an appreciable effect on the hourly elevations of the water surface such as were used in determining the wind effects, reaching a maximum probably not greater than 0.10 foot for any hour. But the determination of wind effects is essentially based on the rate of change of elevation from hour to hour. That rate of change is so slightly affected by the true tides and enters so nearly as an accidental error, positive and negative with almost exactly the same frequency, that the final conclusion as to wind effects is certainly not affected appreciably by the tides.

SEICHES.

A seiche is an oscillation in the waters of the lake under the influence of inertia. It is a free oscillation as distinguished from a forced oscillation. It is a wave motion involving both horizontal transfer of water back and forth and a vertical oscillation of the water surface.

The seiches of Lake Erie, and to a lesser extent of Lake Michigan–Huron, were studied in this investigation because it became evident that they are one of the principal sources of error. It seemed probable that the more clearly the seiches were understood the more effectually could one guard against the occasional abnormally large errors arising from this source.

The seiches were studied principally in three ways:

They were studied by graphs such as are shown on plates 7 to 16. These graphs aided especially in finding the relation of a given seiche to the impulse which started it and in indicating how rapidly the oscillation died out and under what conditions.

The seiches were studied by statistical methods, mainly for the purpose of determining the prevailing periods of oscillation.

The seiches were studied by computing the theoretical period of oscillation of the lake, or of certain parts of it, from the known dimensions of the lake, horizontal distances and depths. This served to establish the probable manner of oscillation by identifying an already determined period as pertaining to a particular manner of oscillation.

SEICHES AT BUFFALO.

At Buffalo 22 days of hourly elevations of the water surface observed at the gage were studied in detail for seiches. These particular 22 days had been especially selected to include periods of unusually large barometric effects and wind effects, especially the latter. It was to be presumed, therefore, that these days included the impulses which started many new seiches of unusual size.

From the observed hourly elevations corrected for wind effects and barometric effects, according to best evidence then available, and the corresponding graphs which had been drawn for all 22 days, a table was made showing every maximum point and every minimum point. The intervals of time from each maximum to the next following maximum and from each minimum to the next following minimum were also tabulated. Each of these intervals is the period of one of the complete oscillations which actually took place about the equilibrium points instantaneously fixed by the wind and barometric effects. Each of these oscillations is probably in general a composite of two or more seiches of different periods and of the forced oscillation produced by the changing winds and changing barometric conditions during that time interval. The immediate problem which confronted the investigator was to detect from a study of 22 days of these composites the seiche periods which prevail at Buffalo.

A frequency table was constructed from the observed periods referred to in the preceding paragraph. It is shown below as table No. 29.

TABLE No. 29.—*Frequency distribution of observed periods of oscillation at Buffalo.*

First.		Second.		Third.	
Length of observed periods in hours.	Number of such periods observed.	Length of observed periods in hours.	Number of such periods observed.	Length of observed periods in hours.	Number of such periods observed.
2	24	6.5	3	11	1
2.5	4	7	6	12	3
3	44	7.5	1	12.5	1
3.5	3	8	4	13	2
4	53	8.5	1	14	1
4.5	2	9	7	14.5	1
5	17	10	3	15.5	1
6	14				

161 periods in first group, mean period = 3.7 hours.
10 periods in third group, mean period = 13.0 hours.
Total number of periods 196.

From a study of this table, combined with a study of the graphs, it appeared that probably the 161 periods included in the first part of the table were all disturbed cases of one prevailing seiche period between 3 and 4 hours in length. Accordingly, the mean of these 161 peroids, namely, 3.7 hours, was taken as a best first approximation to the period of a prevailing seiche at Buffalo.

The periods involving half hours, such as 2.5, 3.5, 4.5, etc., arose from the fact that where two successive hourly ordinates at a maximum or a minimum on the graph were equal, the time of said maximum or minimum was identified as being at the half hour midway between the two hours. Naturally, there were relatively few of these cases as compared with those in which the observed period was an integral number of hours. The frequency-distribution table, No. 29, should therefore be scanned as if these periods involving half hours were distributed equally to the adjacent periods which are in integral hours. The table so scanned shows a maximum frequency at 4 hours.

A study of the graphs showed several cases in which an apparent seiche with a period of about 13 hours stood out clearly with relatively little complication by anything else. With this as a clue, it was decided that possibly the 10 periods included in the third part of the table were all disturbed cases of a seiche with a period of about 13 hours. Accordingly, the mean of these 10 periods, namely, 13.0 hours, was taken as a best first approximation to the period of a prevailing seiche at Buffalo.

Next, the 25 cases in the middle of the frequency-distribution table, No.

29, were inspected in detail on the graphs. This inspection showed evidence in 16 cases out of the 25 that a short-period oscillation (3.7 hours) and a long-period oscillation (13.0 hours) were apparently both in progress at the same time. In five other cases out of the 25 the inspection showed evidence that an oscillation with a period of about 3.7 hours was apparently in progress complicated by other things, but no 13-hour period was evident.

At this point in the investigation it appeared to be probable that the 13-hour seiche was almost or quite continuously present at Buffalo, usually more or less masked by other seiches and by forced oscillations. Accordingly, the principal maxima and minima were selected from among the 196 tabulated in table No. 29 to cover the whole 22 days and to correspond to the supposition of the preceding sentence. From these selected principal maxima a second determination of the long period became available, and, similarly, from the selected principal minima a third determination.

The three determinations of the period of the long Buffalo seiche as thus determined were:

	Period equals
From frequency-distribution table, No. 29, mean of 10 cases	13.0 hrs.
From principal maxima throughout the 22 days, 28 cases	13.4 hrs.
From principal minima throughout the 22 days, 31 cases	12.8 hrs.
Mean	13.1 hrs.

The mean 13.1 hours was adopted as the most probable value of the period of the long seiche, and its uncertainty was estimated at not much more than 0.1 hour.

Again, the graphs were carefully studied in detail for evidence as to apparent damping of the 13.1 hour seiche and for evidence as to the impulses which initiate it. Three conclusions were reached:

(1) Each new large 13.1 hour seiche is preceded by a large discernible impulse from the wind or from barometric pressure. (2) Large 13.1 hour seiches are subject to rapid apparent damping even though some new impulse occurred. (3) A wind impulse, if sufficient to change the elevation of the water surface corresponding to equilibrium by more than 0.20 foot, ordinarily either started a new 13.1 hour seiche or clearly distorted such a seiche which was already in progress.

SEICHES AT CLEVELAND.

At Cleveland 22 days of hourly elevations of the water surface observed at the gage were studied in detail for seiches. These particular 22 days had been especially selected to include periods of unusually large barometric effects and wind effects, especially the latter. It was to be presumed, therefore, that these days included the impulses which started many new seiches of unusual size.

The study at Cleveland was somewhat similar to that indicated above as having been made at Buffalo. The frequency-distribution table for Cleve-

land, analogous to table No. 29, showed clearly a short-period seiche, but furnished no clear evidence of a long-period seiche. There were 252 periods in the table. Of these 252 periods, 165 were not less than 2 hours nor more than 3.5 hours. The mean of these 165 periods was 2.6 hours, which is believed to be the prevailing seiche at Cleveland.

A study of the Cleveland graphs indicated that a long-period seiche might be in existence there much of the time, and yet little or no evidence of it would appear in a frequency-distribution table similar to table No. 29, because the short-period seiche (2.6 hours) was rather persistent and because its shortness of period tended to produce well-defined maxima and minima, which would tend to conceal in such a table any long-period seiche. Accordingly, three determinations of a long seiche period at Cleveland were made by the method of selected principal maxima and minima. The selection was made by somewhat arbitrary rules based on the general proposition that the purpose of the selection was to remove the mask made up of short-period oscillations, mainly 2.6 hour seiches. The first determination was based on one selection of a moderate number of principal maxima and minima and a frequency-distribution table. The second and third determinations were based on a second selection of principal maxima and minima. The three determinations involved, respectively, 29, 31, and 30 identified periods. The three mean periods from the three determinations were in order 13.9 hours, 12.5 hours, and 12.9 hours. The mean of these is 13.1 hours, which is believed to be the period of the prevailing long seiche at Cleveland.

The determination of this period, 13.1 hours, at Cleveland, as indicated above, is weak.

But there are three supporting lines of evidence indicated below, which greatly strengthen the evidence indicated above in favor of a 13.1 period at Cleveland.

First, it is to be noted that the mean, 13.1 hours, of three widely separated values, 13.9, 12.5, and 12.9, agrees exactly with the 13.1 hour period determined independently at Buffalo on the same lake.

Secondly, there were 6 days of the 22 examined at Buffalo and Cleveland which were common to these two stations. On these 6 days the maxima at Cleveland of the supposed 13.1 hour seiche coincided in time, within the errors of identification at the two stations, with the minima at Buffalo of the supposed 13.1 hour seiche there. So, also, the minima at Cleveland coincided closely with the maxima at Buffalo.

Thirdly, the theoretical period of oscillation of Lake Erie lengthwise was computed and found to be 13.1 hours. This, combined with the second line of evidence, clearly identifies the Cleveland and Buffalo 13.1 hour seiches as one and the same thing, as the theory shows that in that case the relation of maxima and minima at the two stations should be that which was actually found.

The theoretical period of oscillation of Lake Erie lengthwise was found by use of the table on page 618 of the Coast and Geodetic Survey Report for

1897. This table is based on the well-known laws of the rate of propagation of a tidal wave, or, in general, of a wave in water of which the depth is small in comparison with the length of the wave. In that case, as shown in the table, the rate of propagation of the wave is purely a function of the depth of the water. For a statement of the theory involved, especially as applied to seiches, consult page 348, in the volume referred to, in the Manual of Tides, part 7, by R. A. Harris.

It was known to the investigator, also on the basis of Dr. Harris's tidal theory, that in a lengthwise oscillation of the lake as a whole the end limits between which the oscillation primarily takes place would probably lie not at the very ends of the lake, but at the point near each end at which the depths begin to change rapidly as the shore is approached.

The mean depth at each part of the lake to be used in computing the rate of propagation of the wave was easily derivable from the estimates of depths already made in connection with the computation of Σx for Lake Erie, as described on pages 44–48.

It was found that the theoretical period of oscillation of Lake Erie lengthwise is 13.1 hours, provided the two end limits of the primary oscillation are at the west end at a meridian about 7,000 feet east of Cedar Point (near Sandusky), and at the east end about 49,000 feet to the westward of the extreme eastern shore, in 60 feet of water, in the meridian which is about midway between Sturgeon Point and Windmill Point. At the eastern limit named the water begins to shoal rapidly. The western limit named is at the entrance of the shallow western extension of the lake, in which the depths are everywhere 6 fathoms or less. These eastern and western limits are shown by three stars on plate 2. In the main portion of Lake Erie (see plate 2) the depths are uniform over the greater portion at 12 to 14 fathoms. There is a relatively small portion in the eastern quarter of the lake in which the depths are from 20 to 34 fathoms.

The evidence is conclusive that the prevailing long-period seiche at both Buffalo and Cleveland is a lengthwise oscillation of Lake Erie between the limits stated in the preceding paragraph. This seiche is of the type which might be called a wash-basin oscillation, in which there is a nodal line which changes but little, if any, in elevation. The water surface falls on one side of the nodal line simultaneously with the rise on the other side.

By a study similar to that indicated above in connection with the 13.1 hour seiche, it appears to be probable that the primary oscillation of the 3.7 hour seiche observed at Buffalo is a lengthwise oscillation in that deep part of Lake Erie already referred to as containing depths of 20 to 34 fathoms, in sharp contrast with the greater part of the lake, which has a very flat bottom in depths of 12 to 14 fathoms. The computation indicated the eastern limit of the 3.7 hour oscillation to be at the same point (marked by three stars on plate 2) as the eastern limit of the 13.1 hour oscillation. The western limit was indicated by the computation to be at the locality marked by two stars on plate 2, in the most constricted part, as to width, of

Lake Erie and where the depth is decreasing very rapidly from east to west.

Similarly, from a study of the depths, the corresponding rates of propagation, and the probable points of reflection, it appears that the 2.6 hour seiche observed at Cleveland is probably a crosswise oscillation back and forth between the 10-fathom curve off Cleveland and the 10-fathom curve on the opposite (Canadian) shore of the lake, about midway between Point Pelee and Point aux Pins.

Though the hourly observations of elevation of water surface at Milwaukee, Harbor Beach, and Mackinaw show that seiches of moderate range are prevalent at each of these stations, it was not feasible to make a careful study of these within the limits of this investigation, with the one exception of the study made for Mackinaw, as indicated below.

SEICHES AT STRAIT OF MACKINAC.

The Strait of Mackinac constitutes a connection between Lake Michigan and Lake Huron. The Strait for a length of 20,000 feet at its narrowest part has a width of about 20,000 feet and a mean depth of about 68 feet. It appeared to the investigator that there was probably a peculiar seiche having its nodal line at about the middle point of the length of the narrowest part of the Strait, and that the oscillation would be such that the whole surface of Lake Michigan would rise while the whole surface of Lake Huron went down, and vice versa, as the current ran alternately westward and eastward through the Strait under the influence of inertia, gravity, and friction, after such an oscillation or seiche had once been started. The investigator did not know of any adequate treatment of the problem of determining the theoretical period of such a seiche. Therefore, he started from the known elements of the problem: the areas of the two lakes; the Chezy formulæ for the relations between the velocity of a steady current flowing in a channel, the slope of the water surface, and the dimensions of the channel; an assumed coefficient of roughness for use in that formula; and the known relations between mass, force, and acceleration. By a step-by-step process he computed the period of oscillation by main strength and computed the probable rate of damping. The conclusions reached were (1) that the period is about 7 hours for a complete oscillation, and (2) that the damping is at least sufficient to reduce the amplitude of the wave by one-sixth part in each successive half wave.

After the computation was complete, an inspection was made of the graph of hourly elevations of the water surface on 42 days at the Mackinaw gage, which is at the eastern end of the Strait. The inspection was not detailed or complete, as the available time was short. Two cases were found on which a 7-hour seiche was apparently started at Mackinaw by an impulse due to barometric pressures, which was peculiarly well adapted to start such a seiche. It was also noted that on May 18–23, 1911, there was

a continuous and unusually large oscillation at the Mackinaw gage, of which the apparent period between successive maxima or minima was either 6, 7, or 8 hours in more than one-third of the cases. The mean period of oscillation for these 6 days was found to be 6.8 hours.

The computation of the theoretical period of oscillation through the Strait of Mackinac needs confirmation, and especially needs a careful study of the possible errors due to certain necessary assumptions made in the computations. So, too, the evidence from the necessarily cursory examination of the graph of hourly elevations at Mackinaw is weak. Nevertheless, the investigator reached the conclusion, subject to the reservations implied in the preceding two sentences, that there is probably a seiche through the Strait of Mackinac of the character indicated, with a nodal line near the mid-length of the narrowest part of the Strait, with a period of about 7 hours.

It should be noted, in closing this general statement in regard to seiches, that as this investigation was based on observed hourly elevations of the water surface it is incapable of detecting seiches having periods less than 1 hour.

The present investigator feels that there is a very interesting field open for further study of the seiches in the Great Lakes, and that rapid progress is possible in this field, which will lead to a much better knowledge than is now available in regard to seiches and tidal oscillations. But he finds, with regret, that he must turn aside, after his little excursion into the field, to carry forward other lines of investigation and of work to which he has already committed himself.

EXAMPLES OF SEICHES.

Plates 14, 15, and 16 show some examples of seiches. All of these plates are drawn to the same scale. On each of them four graphs plotted from hourly ordinates are shown.

The observed hourly elevations of the water surface are shown by the dot-and-dash graph. The elevation of the water surface at each hour referred to mean sea-level may be read in feet from the scale of numbers at the right margin of the plate.

The barometric effect at each hour is shown by the dotted graph. Its amount in feet may be read from the scale of numbers at the left margin of the plate.

The wind effect at each hour is shown by the dashed graph. Its amount in feet may be read from the scale of numbers at the left margin of the plate.

The elevation of the water surface at each hour after correction for barometric effect and for wind effect is shown by the continuous graph. The corrected elevations referred to mean sea-level may be read from the scale of numbers at the right margin of the plate. Each ordinate on this graph is that of the first graph (observed elevations) diminished algebraically by the ordinates of the second (barometric) and third (wind) graphs. This

fourth (continuous) graph shows the oscillations of the water surface, under the influence of inertia, about the instantaneous positions of equilibrium of the water surface fixed by the barometric gradients and the winds.

Consider the information shown on plate 14, pertaining to Buffalo:

Note that the barometric effect, as shown by the dotted graph, was +0.22 foot at 1 a.m. on August 4, 1910, that it increased steadily to +0.46 foot at 8 a.m., then began to decrease at once, and decreased steadily to +0.27 foot at 1 p.m. Thereafter the barometric effect fluctuated but little and slowly until 1 p.m. on August 6. From 1 p.m. on August 6 to 11 a.m. on August 7 the barometric effect decreased slowly and steadily from +0.28 foot to −0.11 foot. Thereafter the barometric effect remained nearly constant.

Note that during these 4 days the wind effect, as shown by the dashed graph, was very small until 5 a.m. on August 4, increased rapidly after that time to +0.32 foot at noon on August 4, then decreased rapidly to +0.01 foot at 9 p.m. on August 4, and thereafter increased slowly to +0.08 at 9 a.m. on August 5. From 9 a.m. on August 5 the wind effect increased rapidly to +0.65 foot at 5 p.m., and then decreased still more rapidly to +0.03 foot at 11 p.m. on August 5. Thereafter, during August 6 and August 7, the wind effect fluctuated but little. It was not more than 0.01 foot at any time between 6 p.m. on August 6 and the end of August 7.

From the graph of observed elevations, note that the water responded to the barometric and wind impluses in the forenoon of August 4 and to the reversed wind impulse of that afternoon. Note that the continuous graph of corrected elevations is very irregular after these impulses, and that during the period 9 p.m. on August 4 to 9 a.m. on August 5, when the water was comparatively free from new impulses, there is some indication of a seiche with a period between 3 and 4 hours and a range of more than 0.2 foot.

Note that the water surface responded to the sudden large upward wind impulse which occurred between 9 a.m. and 5 p.m. of August 5 and to the still larger and more sudden downward wind impulse between 5 p.m. and 11 p.m. on August 5. Note that thereafter, during August 6 and 7, with no large new impluses, the continuous graph (corrected elevations) shows clearly a long-period seiche in progress. The period of this seiche is apparently slightly in excess of 13 hours, as identified from the record of these 2 days. It is believed to be the 13.1 hour seiche for which the evidence is summarized on pages 114–116. Note that the total range of this seiche on August 6 is 0.7 or 0.8 foot and on August 7 is about 0.5 foot. Note its comparatively regular form on August 7.

Examine plate 15, showing the four graphs for Buffalo on October 21–22, 1909. Note that the water surface, as shown by the observed elevations (the dot-and-dash graph), responded to the large and gradual barometric impulse. The most striking thing shown on this plate is the short-period seiche which was evidently in progress throughout the 2 days, continually being modified and distorted but persisting. On the continuous graph

(corrected elevations) there are maxima on October 21 at 4 a.m., 7 a.m., 9 a.m., noon, 6 p.m., 8 p.m., and midnight, and on October 22 at 3 a.m., 7 a.m., 11 a.m., 2 p.m., 7 p.m., and 11 p.m. These apparently define 12 complete waves of the 3.7 hour seiche referred to on page 115. The different values of the period, from the 12 separate intervals between the maxima noted, varied from 2 to 6 hours, with a mean for the 12 waves of 3.6 hours. Similarly, if one judges by the minima on these 2 days, there were 12 complete waves between 3 a.m. of October 21 and 10 p.m. of October 22, no single wave being shorter than 2 hours and none longer than 5 hours. The mean period from these minima is one-twelfth of 43 hours, or 3.6 hours.

Examine plate 16, which shows the four graphs for Buffalo on October 26–27, 1910, and also in the upper left quarter of the plate the four graphs for Cleveland on October 27.

On October 26, 1910, at Buffalo a long-period seiche, apparently the 13.1 hour seiche, was in progress, with a range of 0.5 to 0.8 foot, and the barometric effects and wind effects were not large nor changing very rapidly.

Between 11 p.m. on October 26 and 11 a.m. on October 27 at Buffalo the barometric effect increased very rapidly from +0.01 foot to +0.50 foot. Between 1 a.m. and 10 a.m. on October 27 the wind effect increased very rapidly from −0.07 to +1.02 feet. The two nearly simultaneous impulses, one from the barometric pressures and one from the winds, together tended to raise the water surface at Buffalo by 1.58 feet. Actually, according to the observed elevations, the water rose at Buffalo 4.65 feet — from elevation 571.05 at midnight at the beginning of October 27 to elevation 575.70 at 10 a.m. on October 27 — at the time when the maximum wind effect occurred with a wind of 53 miles per hour from the southwest.

The wind effect at Buffalo decreased very rapidly from +1.02 feet at 10 a.m. on October 27 to +0.14 foot at 4 p.m. on October 27, by which time the wind had died down to 27 miles per hour from the west. Note that the water surface (observed elevations) fell at Buffalo 4.60 feet — from elevation 575.70 at 10 a.m. on October 27 to elevation 571.10 at 7 p.m. From the facts to which attention has been called in this paragraph and the preceding paragraph it is clear that the great rise culminating in the elevation 575.70 at 10 a.m. on October 27 was largely an inertia effect. The barometric effect and wind effect combined, without inertia, would have accounted for only 1.58 feet out of the total rise of 4.65 feet.

That the great rise was largely due to inertia effects and was the first half wave of a new very large seiche is shown by the continuous graph (corrected elevations). It is such a first half wave of a new seiche, produced by a new large impulse, that is believed to produce the occasional excessively large residuals in the daily corrected elevations which are caught and rejected by the criterion for such rejections which is set forth on page 111. Note in table No. 19, page 83, that the residual from the five-day mean for corrected elevations for October 27, 1910, at Buffalo was −0.39 foot,

and that the corrected elevation for this day was automatically rejected by the criterion.

Compare the graphs for Cleveland on October 27, 1910, as shown on plate 16, with those for Buffalo on that plate. Note that the barometric effects and wind effects were much smaller at Cleveland on that day than at Buffalo. Note that the minimum observed elevation occurred at Cleveland at 8 a.m., within 2 hours of the maximum observed elevation at Buffalo, and that the maximum observed elevation occurred at Cleveland at 8 p.m., within 1 hour of being simultaneous with the minimum observed at Buffalo at 7 p.m. The rise of 1.57 feet at Cleveland between 8 a.m. and 8 p.m. corresponded to the fall of 4.60 feet at Buffalo between 10 a.m. and 7 p.m. This is in accordance with the idea that this was largely an inertia effect, the early part of a new seiche affecting the whole of Lake Erie and of the character indicated on page 118. Note that in such a seiche Cleveland and Buffalo water surfaces should normally be changing in opposite directions, and that Cleveland minima should be simultaneous with Buffalo maxima, and vice versa.

GENERALIZATIONS.

In the course of this investigation, during the consideration of the many details of the evidence which it is not feasible to set forth here, and in the efforts to develop an adequate theory, the writer has made certain generalizations in regard to barometric effects, wind effects, and seiches. These generalizations have not been fully established in some parts. Even in such parts as are fully established it is not feasible to set forth the considerations on which they are based, except in part. Yet even under these circumstances it is desirable to give the generalizations as a guide to others, to be used with such degree of caution as may seem best to them. The generalizations are accordingly given in what follows.

GENERALIZATIONS AS TO BAROMETRIC EFFECTS.

Barometric effects upon the elevation of the water surface at a given point are primarily proportional to the distance of said point from the center of gravity of the area of the lake surface. Note the formulæ for R_w and R_n, (17), on page 16.

Barometric effects at a given point are also dependent to a considerable extent upon the shape of the bottom and the configuration of the shore in all parts of the lake. This influence shows in tending to make the P_w and P_n proportionality factors in (19), page 17, depart widely from unity. Note the value of the factors found in this investigation in table No. 16, page 70. It is probable that these proportionality factors tend to be much greater than unity, and the barometric effects correspondingly large, on all lakes having long natural periods of oscillation, say more than 6 hours. If the natural period of oscillation is much less than 6 hours, the water will tend to respond to the rather slowly changing barometric gradients in such

wise as to maintain a moderately close approach to the condition of equilibrium, in which case the proportionality factors would tend to be unity.

The magnitude of the barometric effects on a given lake is dependent upon the position of that lake in regard to prevailing storm tracks. The principal storm tracks for the United States pass near the Great Lakes. Hence, there are frequent cases of large and rapidly changing barometric gradients over these lakes. If lakes of the same shape and size in every respect existed in some tropical region in which fluctuations in barometric gradients were small, as a rule, the barometric effects would be correspondingly small. The barometric effects on Gatun Lake, at the Panama Canal, are probably relatively small, for the reason that the fluctuations in barometric gradients are small.

GENERALIZATIONS AS TO WIND EFFECTS.

The wind effects upon the elevation of the water surface at a given point are dependent to a small extent upon the distance of that point from the nodal line. Note the function which L, the distance, plays in formula (59), page 43.

The wind effect at a given point is dependent largely upon the average value of the inverse cube of the depth, $\frac{1}{D^3}$, between that point and the nodal line. Consult formula (59). This average value is dependent mainly upon the smaller depths involved. Dividing the depth by 10 multiplies $\frac{1}{D^3}$ by 1,000. Hence, in making a first estimate of the probable magnitude of the wind effects at a given point, attention should be riveted mainly upon the shallow portions of the lakes which lie between the point and the nodal line.

The nodal line for each direction of wind tends to be located close to the shallow parts of a lake which is unsymmetrical as to depths. Parts of the nodal line tend to cross the mouths of bays of small depth. Note the positions of the nodal lines on Lake Erie, and the several cases shown in plates 2, 5, and 6, in which detached portions of a nodal line cross bays tributary to Lake Michigan–Huron. There are several other such cases on Lake Michigan–Huron, which could not be shown clearly on the small-scale illustrations of this publication.

The wind effects tend to be very large in bays of small depth, both because of the small depths involved and because of the fact that cross return currents, with a component at right angles to the wind direction, such as are referred to on page 43, which otherwise would hold down the wind effects, are inhibited by land intervening between the bay and the open lake. In making a first rough estimate of the wind effect at a point on a bay it is important to note carefully the extent to which such cross currents are inhibited for winds in each direction.

The magnitude of the wind effects on a given lake probably has little relation to the natural periods of oscillation, or seiche periods, of that lake. The fluctuation in wind impulses imparted to the lake by the surface drift to leeward of the water is so rapid and so erratic, and the surface drift normally remains nearly constant for so short a period, except during very light winds, that the inertia effects, upon an average, probably tend about as frequently to act counter to the wind impulse as to act in cooperation with it. The one exception to the foregoing generalization is that the first half wave of a new extremely large seiche tends frequently to be such as to correspond to an exaggerated wind effect.

GENERALIZATIONS AS TO SEICHES.

The initial impulse or impulses which start a seiche in the Great Lakes are probably much more frequently due to the wind than to the barometric gradients. The wind impulses are in general much more sudden than the barometric impulses. They are certainly much more frequent. The barometric impulses are far from negligible, however, on the Great Lakes. The impulse starting a seiche is clearly traceable to the barometric gradients in some cases. The barometric gradients are probably especially effective in starting the Strait of Mackinac seiches referred to on page 119.

The development or non-development of seiches on a given lake is largely dependent upon the depth of the lake, upon the uniformity of depth, and upon the configuration of the shore. The greater the depth of the lake the smaller will be the rôle played by friction and the smaller the true damping of the seiche. The more uniform the depth over the greater part of the lake the more will the lake tend to act as a single large seiche area and the larger and more persistent will the seiches tend to be. Similarly, the more regular the shore as seen in horizontal projection the more will the lake tend to act primarily as one large seiche area, with correspondingly large and persistent seiches.

Normally, a lake surface has several natural periods of oscillation or seiche periods. Each seiche period and each method of oscillation as a seiche (lengthwise or crosswise) pertains to what may be called a seiche area. Each seiche area is limited either (*a*) by portions of the shore of the lake or (*b*) by a belt in which there is a steep slope in the bottom of the lake separating areas of decidedly different depths. At boundaries of a seiche area of class (*a*) there is no transmission of the oscillation beyond the boundary. There is simply absorption and reflection. At boundaries of a seiche area of class (*b*) there is much more or less effective reflection, depending upon the steepness of the slope of the bottom and the amount of the total change of depth, as well as upon the shape of such boundary as seen in horizontal projection. To the extent that such reflection takes place, the belt acts as a boundary. There is also at boundaries of class (*b*), in addition to absorption of energy due to local movements, a decided tendency for a part of the wave to be transmitted across the boundary into

the next seiche area, where it becomes a forced oscillation for the time being and tends to set up a new seiche of the period natural to that area or to modify such a seiche which may already be there.

In each seiche area the seiche period is dependent upon the depth of the water and the principal dimensions of the area.

The apparent damping of seiches is dependent upon two things: (1) the true damping, due to friction, and (2) the transmission from the seiche area in question into other adjacent seiche areas.

The true damping, due to friction, is of course dependent primarily upon the depth of the water. In deep water the friction is smaller in proportion to the total energy involved in the seiche, and hence the damping is small.

The apparent damping by transmission from the seiche area under consideration into adjacent seiche areas tends, of course, to be greater (1) if the seiche area in question is bounded largely by other seiche areas rather than by shores, (2) if the adjacent seiche, areas are large in comparison with the seiche area under consideration, and (3) if the seiches in progress in the adjacent areas are small relatively to those in the area under consideration. If an adjacent seiche area has in it at a given time a large seiche, there is a tendency for the net transmission of energy across the boundary to be such as to increase the seiche in the area under consideration. In general, there will be a transmission in progress in both directions across the boundary, one tending to dissipate the seiche in the area under consideration and the other to build it up at the expense of the adjacent seiche. In general, if an unusually large seiche is started in one seiche area of several in a lake, the apparent damping in that area will be very rapid at first, until enough energy has been transmitted across to adjacent seiche areas to build sufficiently large seiches there to make the net exchange of energy across the seiche area boundaries nearly zero. Thereafter the apparent damping will be mainly true damping, due to friction, and therefore relatively slow.

From the foregoing it is clear that if a lake has a compact area with regular shores, has nearly uniform depths, and has these matters so related that the whole lake is composed of only one or a few seiche areas, large and persistent seiches will occur in that lake. On the other hand, if a lake has a very irregular shore and a straggling area, if its depth varies greatly in different parts, and if these matters are so related that the whole lake is broken up into many seiche areas none of which predominates in size over the others, the seiches in that lake will in general be small, and each new unusually large seiche will be rather promptly reduced to moderate size by transfer of energy across seiche-area boundaries.

Consider the contrast between Lake Erie and Lake Michigan–Huron in the characteristics indicated in the preceding paragraph. Use plates 2, 5, and 6 in visualizing the contrast.

As a whole, Lake Erie is comparatively compact, with regular shores. Over more than one-half of its area the bottom is almost as flat as a floor, with depths varying only from 11 to 14 fathoms for 120 miles along its axis.

It is broken up into only a few seiche areas. There are probably only three important seiche areas in Lake Erie: (1) the area of depth greater than 20 fathoms in the eastern part of the lake; (2) the continuous area of more than 10 fathoms depth extending from a point near the eastern end of the lake (marked by three stars on plate 2) nearly to Sandusky and Point Pelee, near the western end of the lake; and (3) the part of the western end of the lake which is largely cut off from the main lake by Point Pelee and a chain of islands including Pelee Island and Kelley's Island. The first-mentioned seiche area is included within the second-mentioned.

In contrast, consider Lake Michigan–Huron. It has a straggling area bounded by extremely irregular shores interrupted once by the whole of the lower peninsula of Michigan and again by the peninsula and islands which in part cut off Georgian Bay and North Channel from the main portion of the lake. Lake Michigan–Huron is probably broken up into at least 8 seiche areas, which are all of primary importance. The writer estimates these areas, by examination of the Lake Survey charts, to be as enumerated below:

(1) That part of Lake Michigan south of latitude 44°, in which the depth is more than 50 fathoms and less than 90. Note that the regularity of this area is broken in its middle portion by a small oval, within which the depth is more than 90 fathoms, and another oval near it, within which the depth is less than 50 fathoms.

(2) That northern part of Lake Michigan extending from a point north of latitude 44° nearly as far north as the main entrance to Green Bay, in which the depth is more than 90 fathoms.

(3) Green Bay.

(4) That northeastern part of Lake Huron which is more than 50 fathoms deep, extending in a long area of gradually increasing width from latitude 46° and longitude 84° southeastward to latitude $44\frac{1}{4}$° and longitude 82°.

(5) North Channel, the long narrow area at the extreme north end of Lake Huron, separated from the main portion of the lake by Manitoulin Island.

(6) Georgian Bay.

(7) The unnamed bay which is a southern extension of Lake Huron and which terminates at Port Huron and the entrance of the St. Claire River.

(8) Saginaw Bay.

So far as there is a 7-hour seiche through the Strait of Mackinac, as discussed on pages 119–120, the whole of Lake Michigan–Huron is acting as a single seiche area.

In accordance with the contrast between Lake Erie and Lake Michigan–Huron, to which attention has just been called, the observations indicate that the seiches on Lake Erie are much larger in range, as a rule, than those on Lake Michigan–Huron; that the Lake Erie seiches persist with a relatively large range for several days at a time; and that on Lake Michigan–Huron, though very large seiches sometimes occur, such seiches are apparently damped down to a small range, as a rule, after only a few oscillations. The prevailing condition at each gage on Lake Michigan–Huron seems to be that small seiches are in progress, seldom disappearing completely and seldom large and regular enough to make it easy to detect the seiche periods.

This last statement is cautiously made, because the limits of this investigation have made it impossible to study the seiches in Lake Michigan–Huron with the same care that was given to Lake Erie seiches.

GENERALIZATION AS TO PREVAILING CONDITIONS ON THE GREAT LAKES.

Apparently, if one is to appreciate fully the meaning of the continuous record from a gage recording elevations of the water surface at any place in the Great Lakes, he must have the general conception stated in the following long paragraph, and must appreciate that the gage is in one of the several seiche areas of that lake.

At the beginning of any hour on any one of the Great Lakes there are one or more seiches in progress in each of the several seiche areas of that lake, and all oscillations (seiches) are taking place about a certain equilibrium surface which is not horizontal but which has a certain position and slopes fixed by the winds and barometric gradients operating on the lake at that instant. During that hour changes occur in the direction and velocity of the winds and in the barometric gradients. These changes produce corresponding changes in the position and slopes of the equilibrium surface about which all oscillations due to inertia (seiches) take place. The changes in the equilibrium surface produce new forced oscillations in every seiche area. The forced oscillations develop new seiches with the natural period of each seiche area, which new seiches, being superposed on the seiches already there, modify them, sometimes mainly in phase, sometimes to partially stop them, but as a rule to increase their range. From each seiche area forced oscillations are sent continuously into adjacent seiche areas, tending to develop in these areas new seiches, which are superposed on the old, as indicated in the preceding sentence. In general, whenever a change occurs in the equilibrium surface — when there is a new set of wind and barometric impulses over a lake — there will be a considerably larger seiche produced in some one of the various seiche areas than in the others. This may come about because (*a*) the new impulse happens to be timed to correspond in the period of its change to the natural period of that seiche area, or (*b*) the new impulse may happen to reinforce a particular wave of a seiche in progress in that area and so to decidedly increase its range, or (*c*) the forced oscillations sent in from the adjacent areas may happen to coincide with each other and with a seiche in progress and so to decidedly increase its range. The exchange of energy between different seiche areas will tend to produce a very much greater apparent damping of this particular large seiche than of the other seiches in progress and tend to bring about a normal distribution of seiche range to the different areas. As time progresses there will be continual modification of seiches by new wind and barometric impulses, usually tending toward an increase in range of the seiches. The true damping by friction, on the other hand, tends continually to reduce the range of the seiches. The net effect of these two counter in-

fluences on seiche range is to cause the seiche range in general to increase during periods of increasing intensity of wind and barometric impulses, to decrease rather rapidly during periods of decreasing intensity of wind and barometric impulses, and still to decrease but slowly during periods when such intensity is approximately constant. One must see all these things to appreciate fully the meaning of a gage record, in addition to seeing in it the fluctuations of elevation of the mean lake surface due to inflow from the lake above, outflow to the lake below, run-off from the surrounding land, rainfall on the lake surface, and evaporation from the lake surface.

POSSIBLE APPLICATIONS OF RESULTS OF THIS INVESTIGATION.

The outcome of this investigation is a more accurate knowledge than was heretofore available of the wind effects and the barometric effects upon the elevation of the water surface at any given point on the Great Lakes or on any free water surface, and some increase in the available knowledge of seiches and of their relation to accurate determinations of the mean elevation of the water surface from a given gage record. Consider, now, very briefly, some of the possible applications of the results of this investigation.

APPLICATION TO A STUDY OF LAWS OF EVAPORATION.

As stated on pages 1–4, the investigation here reported upon is a part of a larger investigation having for its purpose determining the laws of evaporation from large water surfaces, such as the surfaces of lakes and rivers. To attain this end, it is proposed to consider each of the Great Lakes in turn as an evaporation pan and to evaluate from day to day (1) the change of content, (2) the income, and (3) the outgo, including evaporation. To accomplish the purpose it is necessary to evaluate the change of content from day to day with great accuracy. That change of content is measured by the change in elevation from day to day of the mean surface of the whole lake. Each recording gage measures the change in elevation of the surface of the lake at the point at which the gage is located. Formerly the only feasible way to fix the elevation of the mean surface of the lake was to take it as equal to the mean of the elevations at the two or three points on the lake at which first-class gages were operating. Now, as an outcome of this investigation, the elevation of the mean surface of the whole lake may be determined on any day by applying the known corrections for wind effects and barometric effects at any gage to the observed elevation for that gage. The proper weighted mean for the several gages may be taken and the few abnormal values may be detected and rejected by a definite criterion. In the place, then, of the former values of mean elevation of the whole lake surface, of a certain degree of accuracy, one has now available values of a much higher degree of accuracy. The increase in accuracy, which operates to increase the possible accuracy of the evaporation investigation, may be seen (*a*) by comparison of the two sets of residuals in tables Nos. 19 to 25,

(*b*) in the probable errors shown in table No. 28, page 110, and (*c*) in the discussion of these probable errors on pages 111–113. It appears that from observations at Mackinaw alone the mean elevation of the whole of Lake Michigan–Huron on any day may possibly be determined with a probable error less than ±0.010 foot. From the three stations Milwaukee, Harbor Beach, and Mackinaw together, this probable error may be reduced still more. It appears that the change in elevation of the mean surface of the whole of Lake Michigan–Huron in one day may possibly be determined with a probable error less than ±0.007 foot — an accuracy hitherto unattainable.

The evaporation from Lake Michigan–Huron in a single day, measured in depth of water taken off the whole surface of the lake, probably varies between limits which are approximately 0.000 foot and 0.021 foot. The new accuracy now attainable in determining the fluctuation in the elevation of the mean surface of the lake from day to day evidently makes the proposed method of measuring the small variations in evaporation much more powerful than it otherwise would be. Heretofore the method seemed to be barely within the range of possibility. Now it seems to be certain that the method will succeed. It certainly will succeed so far as success hinges on the evaluation from day to day of the total content of the lake.

APPLICATION TO REGULATION OF THE GREAT LAKES.

The vision is gradually taking form that the elevation of the water surface of each of the Great Lakes must be regulated by movable dams, or the equivalent, for the benefit of navigation, for the benefit to be derived in connection with the development of power in hydro-electric stations on the connecting streams, and to provide for the increased use of the water from Lake Michigan–Huron for sanitary purposes at Chicago. It is now clear that the return to the people of the United States from such regulation would be many times its cost.

When that regulation becomes a reality, as it certainly will in due time, it will then become important to detect each fluctuation in elevation of each lake surface as soon as possible after it occurs and as accurately as possible. Each fluctuation must be detected soon after it occurs in order that the desirable change in regulation may be made. It must be detected accurately, because the total range within which the regulation must operate is small, and the means of regulation, the movable dams or the equivalent, will produce but very slow alterations in lake elevations. The quantities to be dealt with, for the desired valuable regulation, are hundredths of feet of elevation rather than tenths. The promptness and accuracy with which all fluctuations may be recognized is greatly increased by the outcome of this investigation. This is indicated in part by the comments on accuracy made in connection with the proposed application to the study of evaporation. It may be emphasized from another point of view by noting that there are

frequent periods of 10 days or more on which the observed elevations at a gage are continuously too high or continuously too low to represent the elevation of the mean lake surface. As extreme cases, note the facts for the Cleveland gage in August 1910 and for the Milwaukee gage in July 1911.

At Cleveland (see table No. 20, page 85, table No. 26, page 108, and plates 8 and 9), in August 1910 the correction for barometric effect was + on 25 days out of 31. The maximum correction for barometric effect was +0.24 foot on August 25. On the 5 days August 21–25 the average correction was +0.18 foot. For the whole month of August the corrected elevation was 0.06 foot greater than the observed.

At Milwaukee (see table No. 21, page 88, table No. 27, page 109, and plates 11 and 12), in July 1911 the correction for barometric effect was + on 28 days out of 31. The maximum correction for barometric effect was +0.30 foot on July 25. On the 7 days July 20–26 the average correction was +0.15 foot. For the whole month of July 1911 the corrected elevation was 0.09 foot greater than the observed.

The results of this investigation, in the form of corrections for wind effects and barometric effects, and detection of abnormal values by certain criteria, should be applied to regulation of the Great Lakes, (*a*) directly, to furnish a more accurate knowledge than would otherwise be possible, day by day, of the actual mean elevation of the whole surface of each lake, and therefore of the total water content of the lake, and, (*b*) indirectly, by enabling the laws of evaporation and of run-off into the lake to be determined and understood in such wise that a forecast of the total yield of water to each lake for weeks and possibly months in advance may be made with much greater accuracy than would otherwise be possible.

APPLICATION TO DETERMINATION OF MEAN SEA-LEVEL AND TO PRECISE LEVELING.

Elevations determined by precise leveling are referred to mean sea-level by means of observations taken at tide gages. The mean sea-level as fixed by the observations at a given gage is in error by an amount dependent on the configuration of the shores and the bottom in the surrounding region and upon the prevailing winds. The method and constants now available will enable one to compute the necessary correction to be applied to the observed mean sea-level to eliminate the wind effect and so to obtain the true mean sea-level. Such a necessary correction may be small at certain gages. It is important to prove it to be small in such cases. The corrections are probably large enough at some gages to predominate over the accumulated errors in the precise leveling for hundreds of miles from the gages.

The corrections for barometric effect should be applied similarly in connection with the determination of mean sea-level.

A concrete idea of the possible errors inherent in the present determinations of mean sea-level and their relation to the errors of precise leveling may be secured as follows: Note that at the bottom of tables Nos. 26 and 27 the mean elevation as observed for a whole season agrees within 0.03 foot with the corrected elevation for the season in every case except at Buffalo. At Buffalo, however, the mean of the observed elevations for the three months of 1909 differs by 0.11 foot from the mean of the corrected elevations, and for the 5 months of 1910 the difference is 0.10 foot. In other words, the omission of the wind and barometric corrections at Buffalo leaves an avoidable error of 0.10 foot in the derived elevation of the water surface from the 5 months of observation.

The probable error of a difference of elevation of two points, determined by the precise leveling of the Coast and Geodetic Survey, is about ± 0.7 millimeter into the square root of the distance between the two points in kilometers measured along the line of leveling. In other words, an error of 0.03 foot $=9.1$ millimeters has an even chance of occurring in 169 kilometers $=105$ miles of leveling from a tide gage. The error of 0.10 foot referred to above in connection with the 5 months observation at Buffalo has an even chance of occurring in about 1,800 kilometers $=1{,}100$ miles of precise leveling. This statement neglects the possible small systematic errors in the leveling and also the strengthening of the level line by its connection with a net of level lines.

Judging from a study of the charts and his present knowledge of the laws controlling wind effects, the writer estimates that the wind effects are as persistently large and of one sign at Cedar Keys, Florida, as at Buffalo. One of the gages at which mean sea-level was determined for use in connection with precise leveling was at Cedar Keys. Similarly, the writer estimates that at Sandy Hook, New Jersey, near New York City, the chance of error due to the omission of corrections for wind effects and barometric effects is not much less than that at Buffalo and is certainly greater than that at the other four stations of tables Nos. 26 and 27. Hence, even though mean sea-level is determined from an average of several years of observation at Sandy Hook, it is possibly in error by an amount exceeding the error accumulated in the precise leveling over hundreds of miles. It should not be overlooked in this connection that the prevailing winds and the prevailing barometric gradients tend to be seasonal, to be repeated each year, and that therefore the taking of a mean for several years is of only moderate effectiveness in reducing the error in the mean. The monthly values of mean sea-level at various tide gages support the statement by showing a seasonal variation, as a rule, and thereby incidentally indicating that the wind effects and barometric effects are certainly decidedly appreciable in the monthly means.

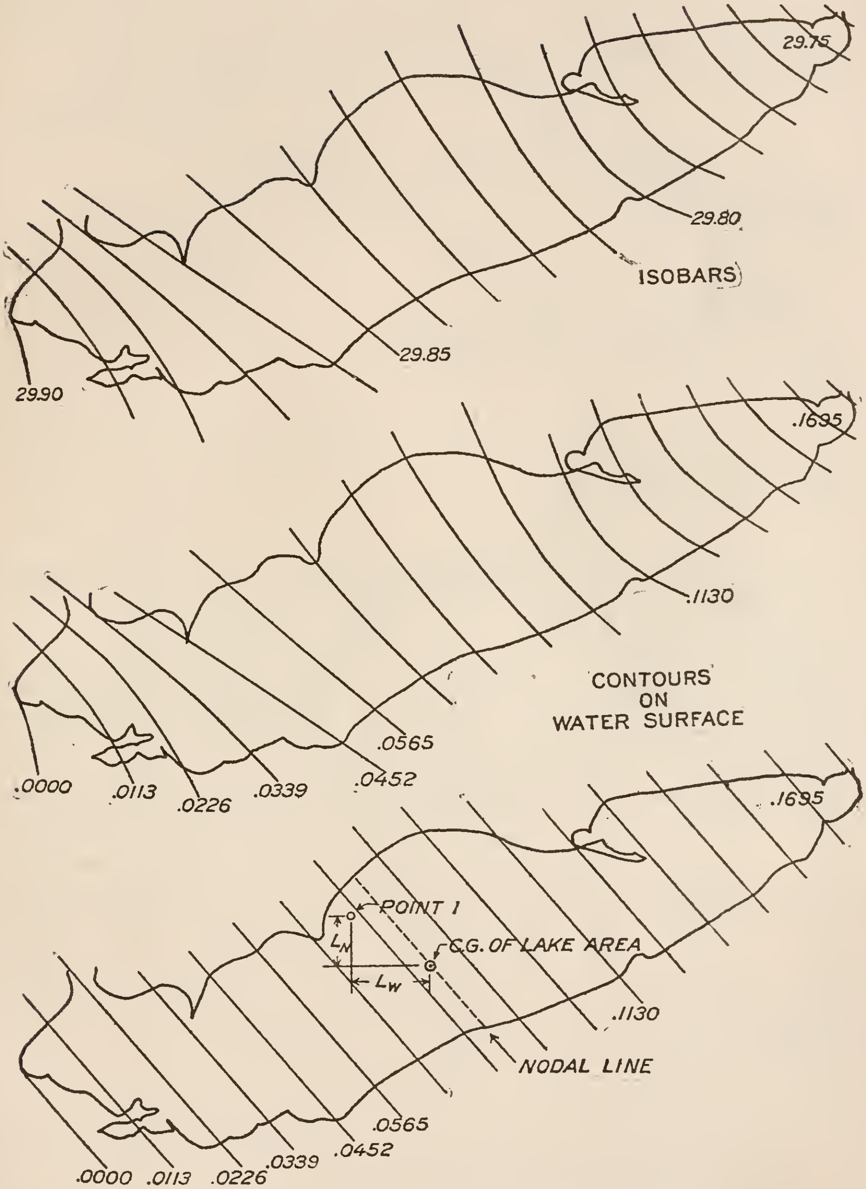
APPLICATION TO DETERMINATION OF TILTING OF THE GREAT LAKES REGION.

From the evidence derived from gages operated over long periods or during widely separated years, at various points of Lake Michigan–Huron, the eminent geologist, G. K. Gilbert, determined that the whole region covered by this lake is slowly tilting to the southwestward, and secured a determination of the rate of tilting. This determination would obviously be strengthened if the corrections for wind effects and for barometric effects at the gages were applied by using the methods and constants made available by this publication. This investigation by Gilbert was published as a part of the Annual Report of the U. S. Geological Survey for 1896–97, part II, pages 595–647, under the title Recent Earth Movement in the Great Lakes Region.

The rate of tilting as derived was .0042 foot per mile per century — an exceedingly small rate of change. The conclusion was derived from apparent changes of relative elevation of the water surface as measured at different gages on Lakes Michigan–Huron, Erie, and Ontario in different years. The amounts of change involved are of the order of 0.1 to 0.2 foot in a period of 20 to 40 years. Evidently, when such small changes are in question there is more chance of securing the necessary accuracy if corrections as large as those shown in tables Nos. 19 to 23, pages 80–96 of this publication, for barometric effects and wind effects, are taken into account. Gilbert limited his deductions largely to days on which there was little wind. But an inspection of the computations made in connection with the present investigation shows that the barometric corrections are by no means negligible on the Great Lakes on days of little wind. Gilbert himself saw the desirability of such corrections, as shown by various statements in his paper.

The deductions of Gilbert are probably correct in the main. But a new investigation based on observed elevations of water surface corrected for wind effects and barometric effects would have greater accuracy and is desirable. Such a new investigation might be in part a recomputation from the data which were used by Gilbert. Much new data of a high degree of accuracy, in the form of observations made in the 24 years since Gilbert's paper was written, are also available.

PLATE 1

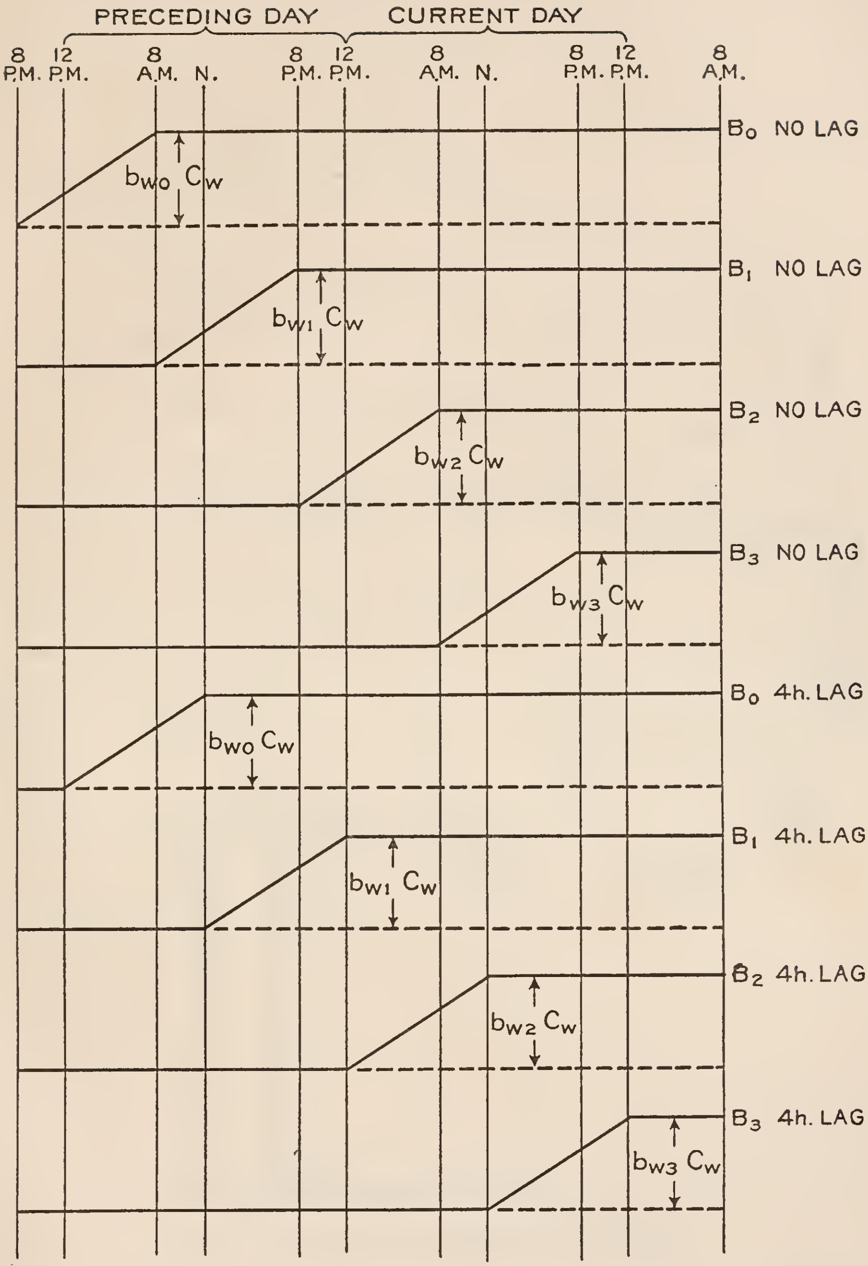


Isobars and contours on water surface of Lake Erie, August 5, 1910.

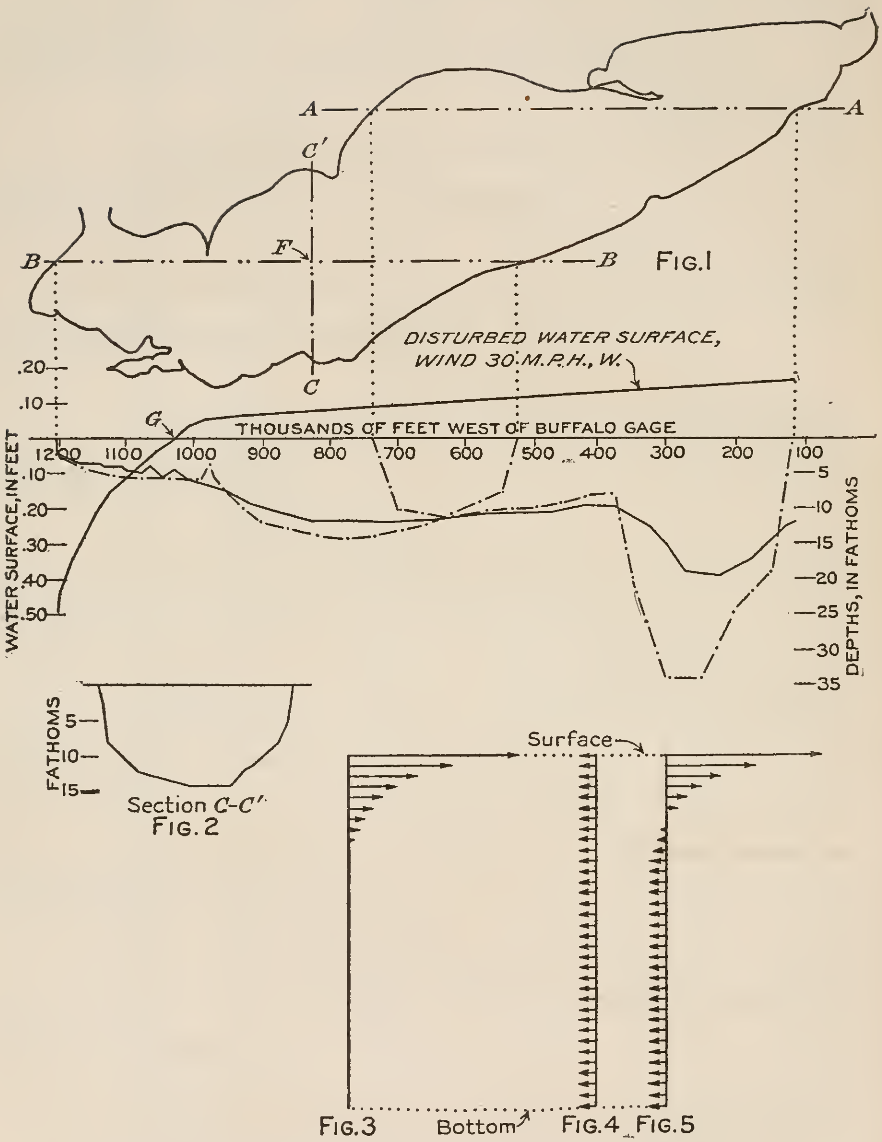
Plate 2

Nodal lines and depths in Lake Erie; barometric points around the Great Lakes.

PLATE 3

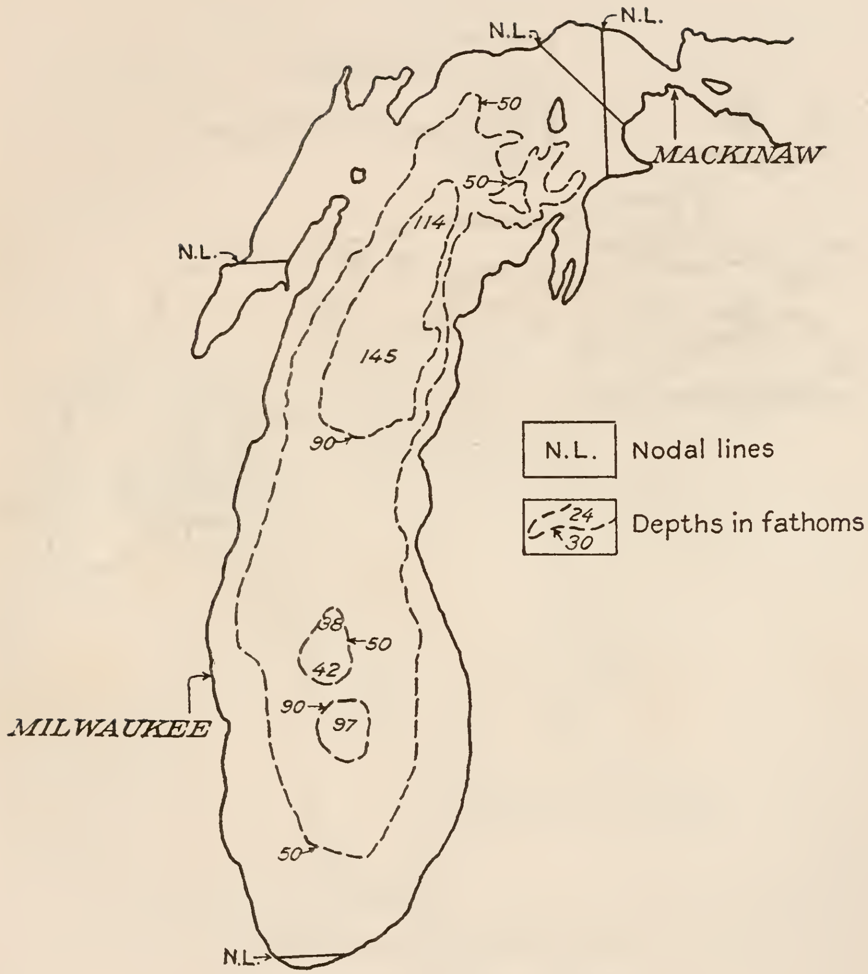


Relation of barometric change to rise of water surface. See text under subheading, "Form of Observation Equations for Barometric Effects."



Wind effects, disturbed water surface, and currents.

Plate 5

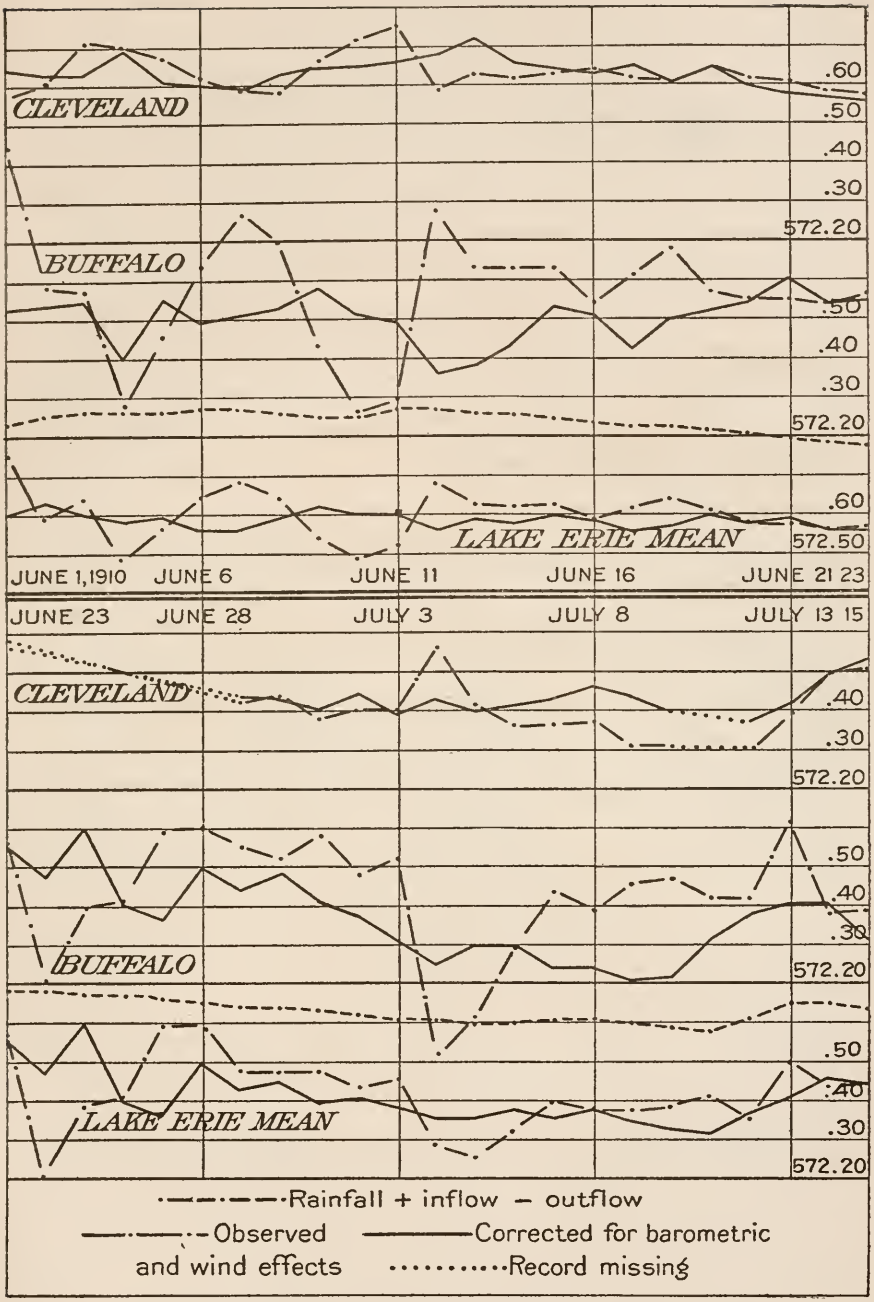


Nodal lines and depths in Lake Michigan.

Plate 6

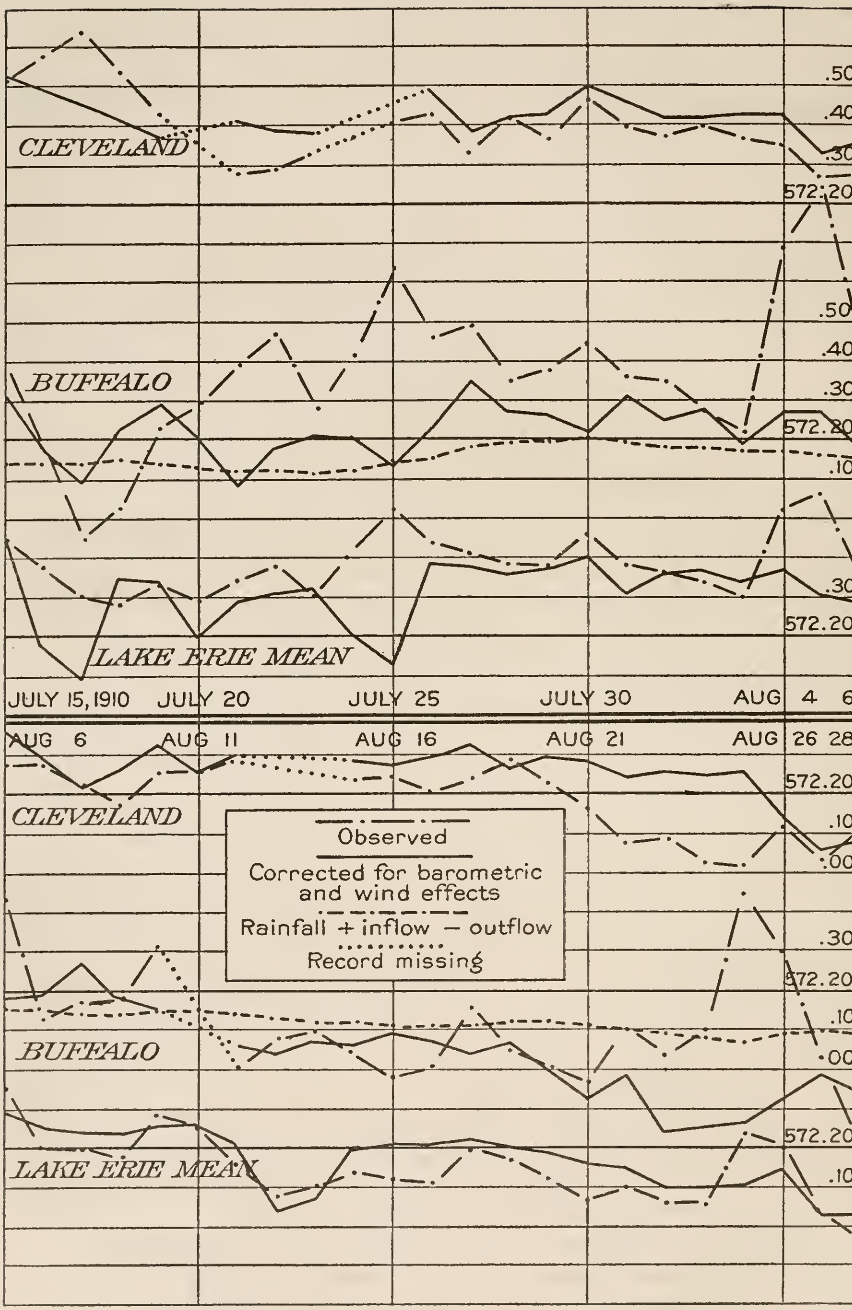
Nodal lines and depths in Lake Huron.

Plate 7



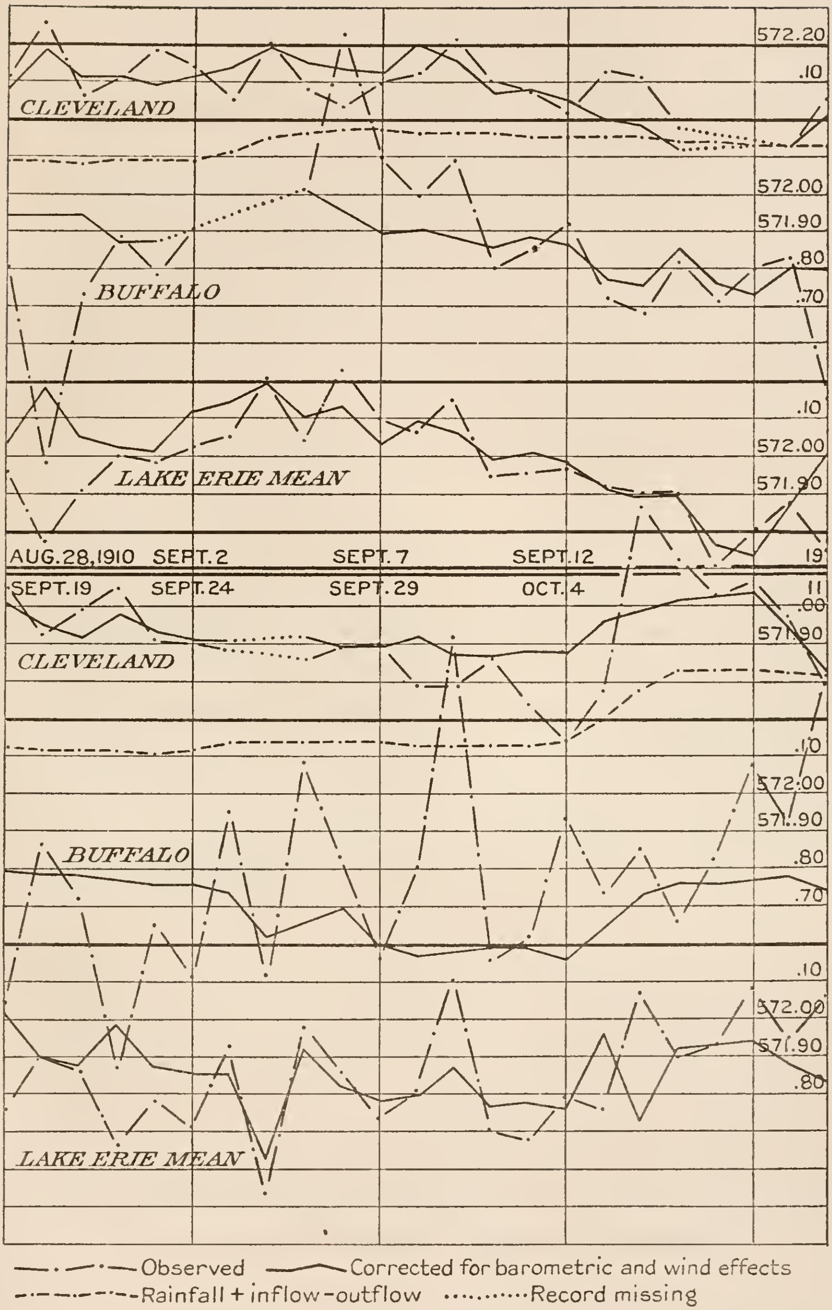
Elevations of water surface, Lake Erie, June 1–July 15, 1910.

PLATE 8



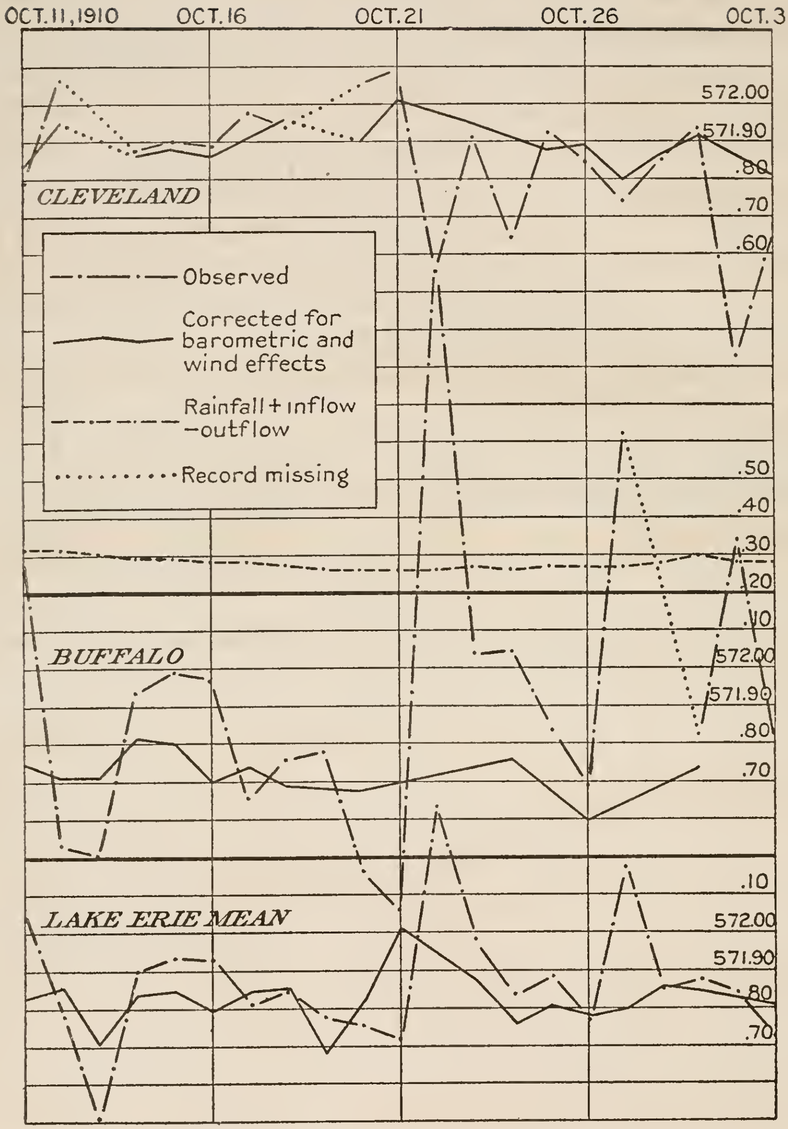
Elevations of water surface, Lake Erie, July 15–August 28, 1910.

PLATE 9



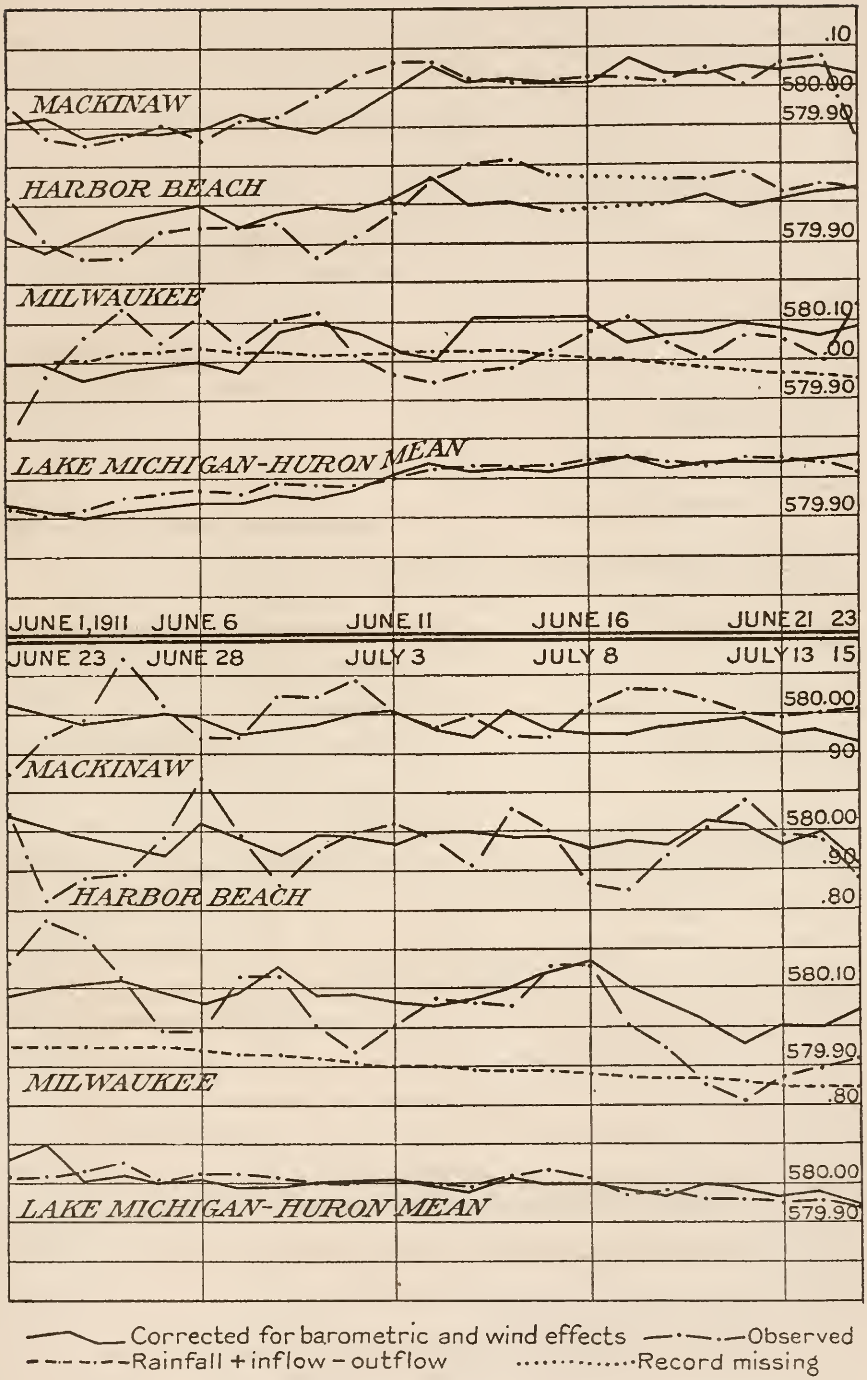
Elevations of water surface, Lake Erie, August 28–October 11, 1910.

Plate 10

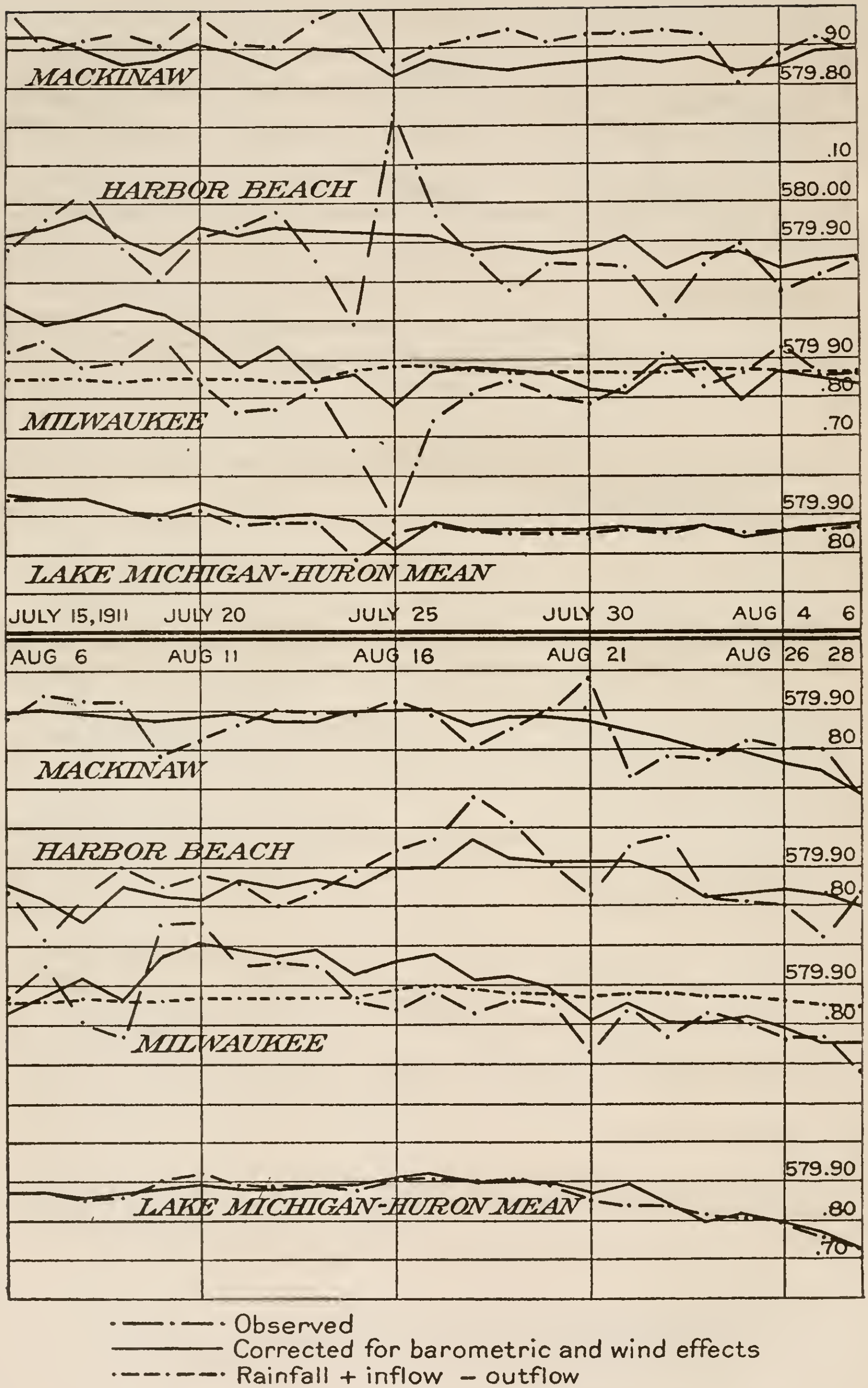


Elevations of water surface, Lake Erie, October 11–31, 1910.

PLATE 11

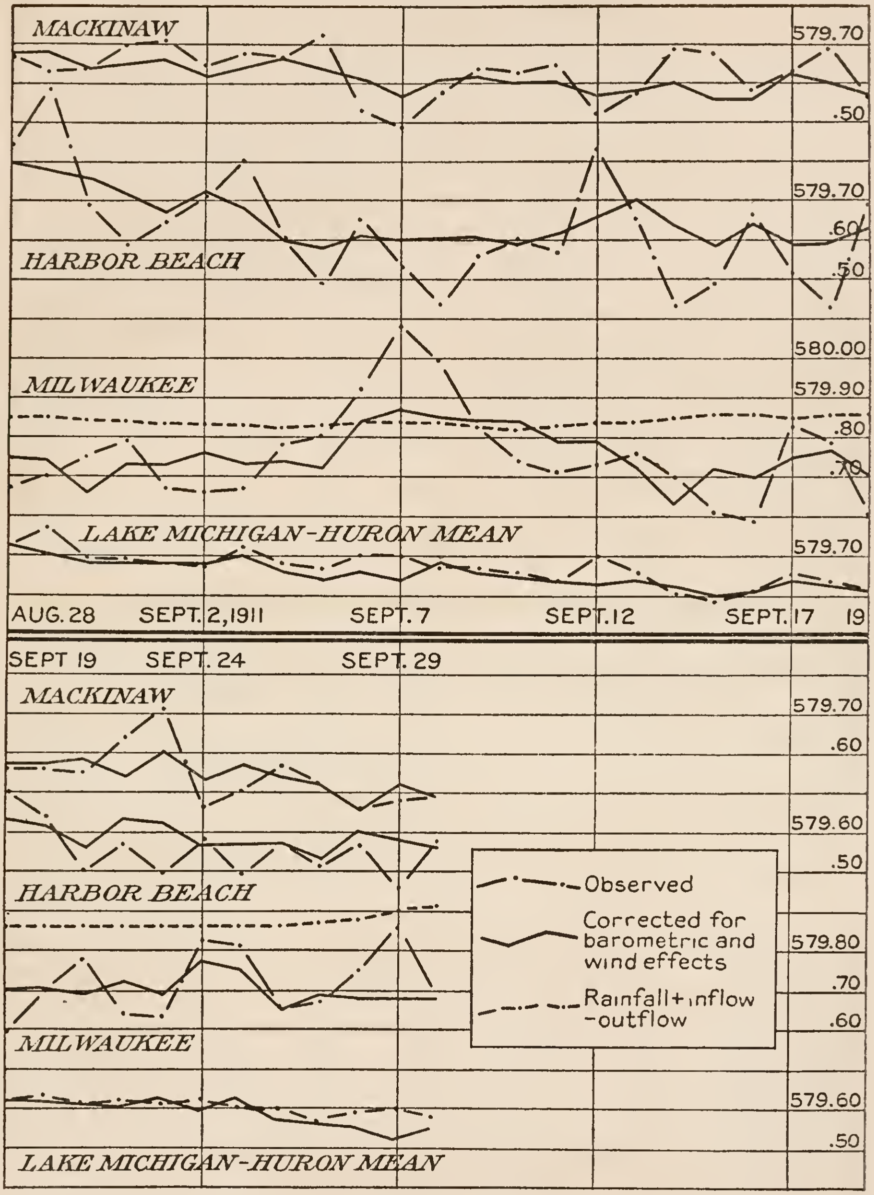


Elevations of water surface, Lake Michigan-Huron, June 1–July 15, 1911.

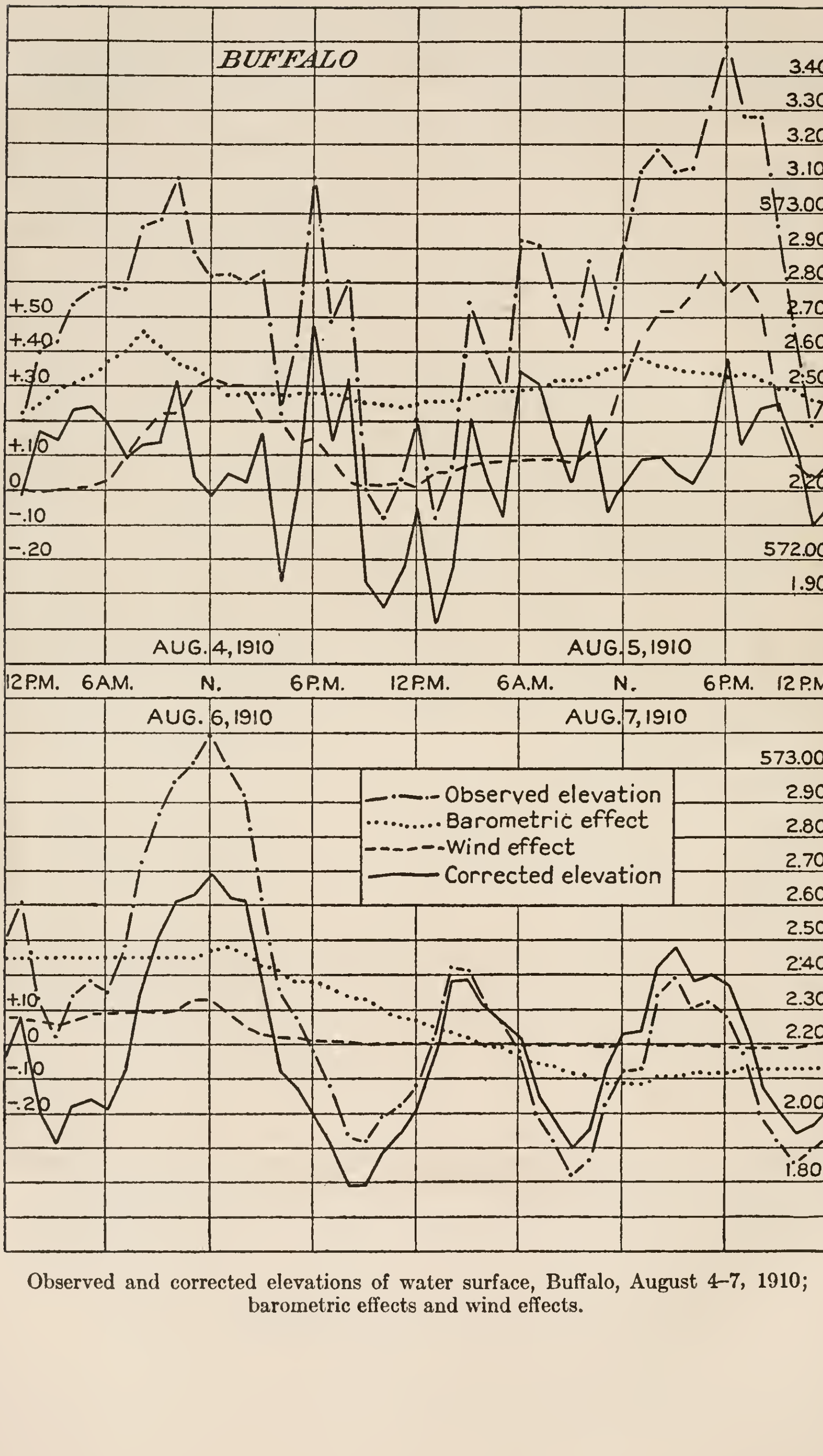


Elevations of water surface, Lake Michigan–Huron, July 15–August 28, 1911.

PLATE 13

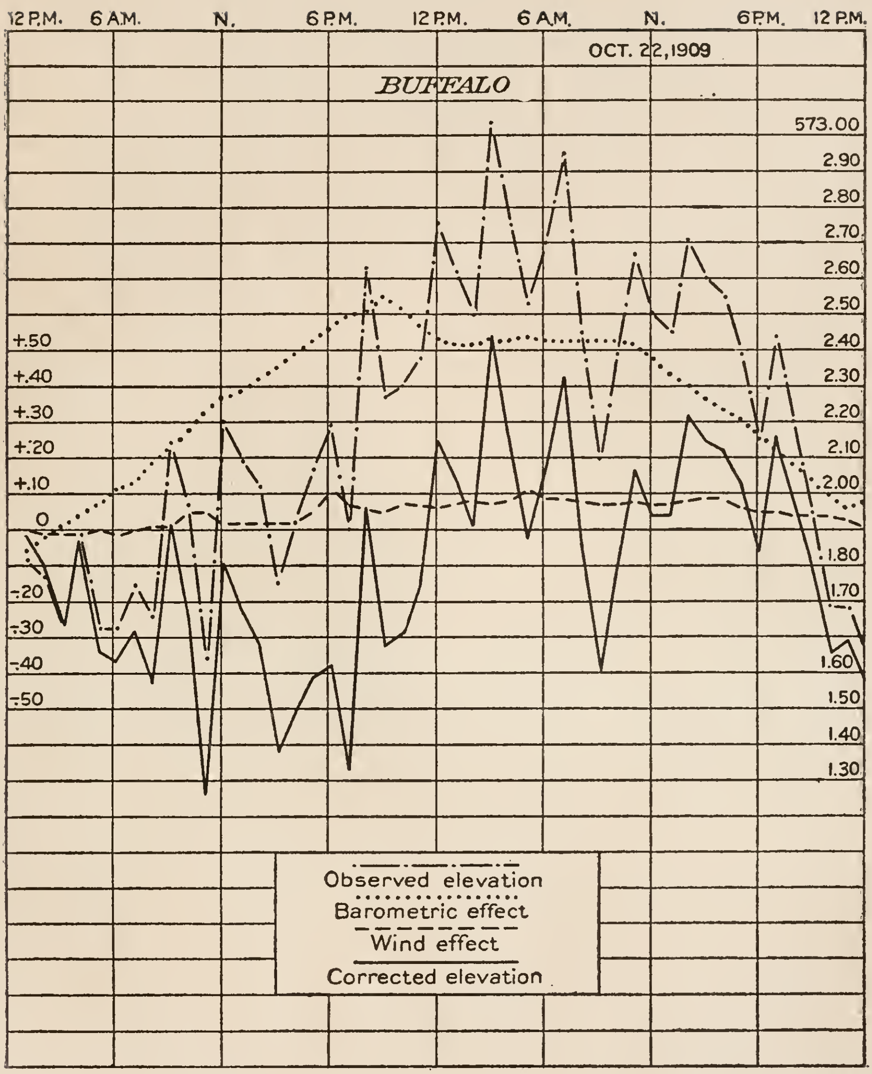


Elevations of water surface, Lake Michigan–Huron, August 28–September 30, 1911.



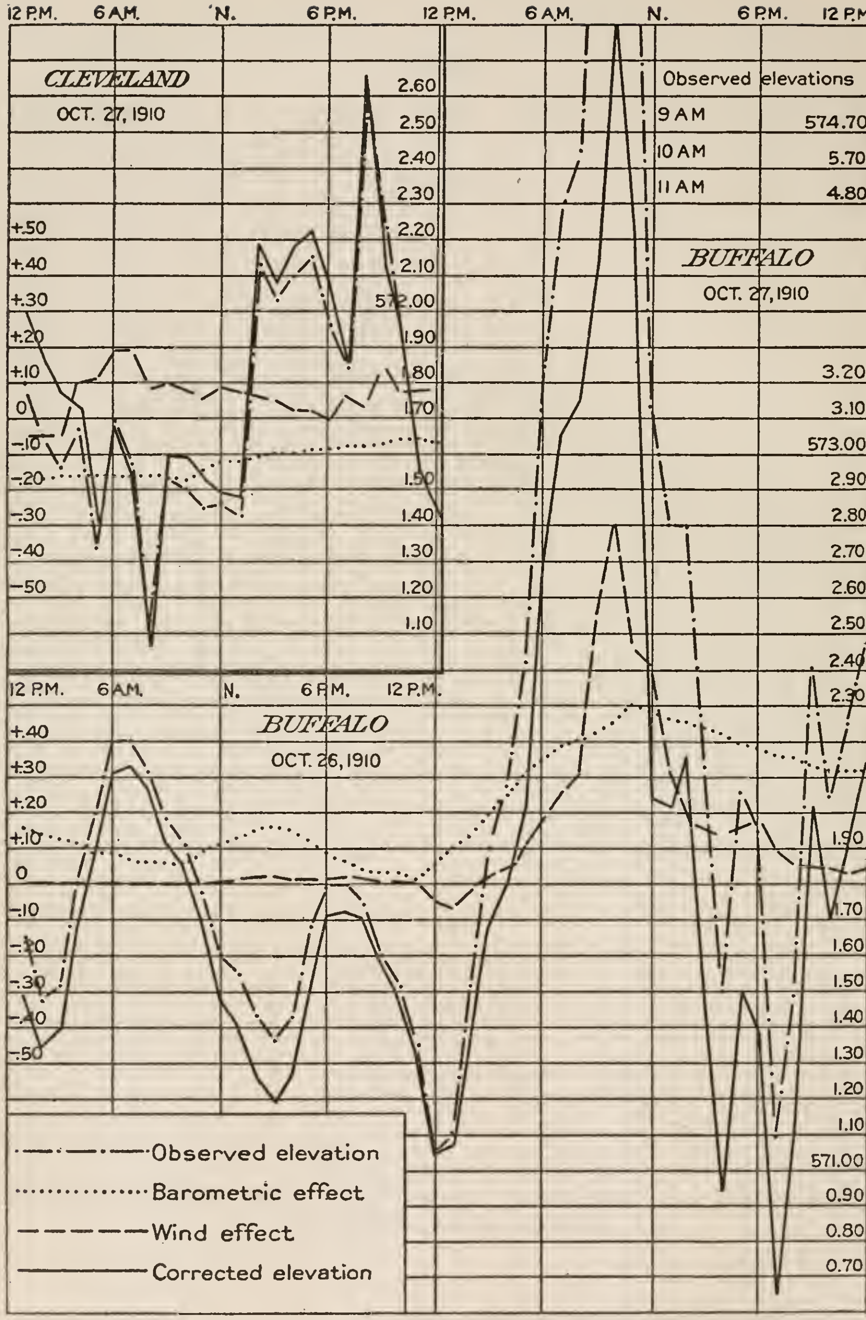
Observed and corrected elevations of water surface, Buffalo, August 4–7, 1910; barometric effects and wind effects.

PLATE 15



Observed and corrected elevations of water surface, Buffalo, October 21–22, 1909; barometric effects and wind effects.

PLATE 16



Observed and corrected elevations of water surface, Buffalo, October 26–27, 1910, and Cleveland, October 27, 1910; also barometric effects and wind effects.

EFFECTS OF WINDS AND OF BAROMETRIC PRESSURES ON THE GREAT LAKES

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